

MIT SLOAN SCHOOL OF MANAGEMENT

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Problem Set 5: Options
Not due

1. You are given the following prices:

Security	Maturity (years)	Strike	Price (\$)
JK stock	-	-	94
Put on JK stock	1	80	5
Call on JK	1	80	?
Tbill (FV=100)	1	-	91

- (a) What is the price of the call option?
 - (b) Now suppose that the market price of a JK call option with strike 80 was 30. When you tell your boss that the price you found in part (1) is different, he tells you that you must be wrong, since the market is efficient and thus always right. In other words, the formula you have used to price the call option is just a theory, but in practice it must be that the market knows something that you don't, perhaps that the stock price is expected to go really high. Do you agree or disagree with him?
 - i. if yes, can you strengthen his argument?
 - ii. if no, can you prove him wrong?
2. In the industry, trader often decomposes the value of an option into "time value" and "intrinsic value". The intrinsic value of an option is the value of the option if you exercise it now, whereas the time value of the option is the difference between the price and the intrinsic value of the option.
- (a) Given a call option on a stock trading at price S , and strike of K , what is the intrinsic value of the stock?
 - (b) For call options of the same strike, how does the time value changes with maturity? Explain (intuition suffices)
 - (c) For call options of the same maturity, how does the time value changes with the strike? Explain (intuition suffices)
 - (d) Stocks of company A and B are trading at the same price. There are two call options of exact same strike and maturity. Is it necessary that the two options trade at the same price? How about the time value of the two options?

3. You are asked to price some options on ABC stock. ABC's stock price can go up by 15 percent every year, or down by 5 percent. Both outcomes are equally likely. The risk free rate is 5 percent, and the current stock price of ABC is 100.
- Find the risk neutral probabilities
 - Price a European Put option on ABC, with strike 100 and maturity 3 years. What is the replicating portfolio?
 - Use the Black-Scholes formula to price the option, using a standard deviation of 10 percent per year. Contrast the result to your answer above. (Note: the standard deviation of stock return is 10 percent in this case)
 - What is the price of an American option with the same characteristics?
 - Now suppose that the stock pays a dividend of 20 right after the beginning of year 2. Repeat the calculations you did at (2) and (4). Has anything changed?
 - After your calculations, an "anonymous" trader calls you up and tells you that he thinks that ACC put options are a really good deal, because he has some inside information (which will be revealed in a couple of days), that the probability of ABC going down is actually 80%. Yet at this point you remember that the 407 class taught you that you priced options using the *risk-neutral*, instead of the actual probabilities. What do you reply to the trader (except that insider trading is illegal, of course) ?
4. Consider a stock that pays no dividends on which a futures contract, a call option, and a put option trade. The maturity date for all three contracts is the same, the exercise price for both the put and the call is K , and the futures price is F . Show that if $K = F$, then the call price equals the put price.
5. (Hedging with options) Since option payoffs are correlated with the price of the stock, one can use an option to hedge the risks associated with the stock. This questions ask you how to use a put to hedge the price risk of a stock, and more.
- Give two reasons why using option to hedge the risks of stock is easier than using forwards or futures, especially for small individual investors.
 - To hedge the downside risk of a stock, what should you do? Assume you hold 1000 contracts of the stock.
 - Suppose $S = 31$, $K = 30$, $\sigma = 20\%$, $T = 1$ year, $r = 5\%$ per year. What is the fair price of your hedge?
 - Suppose you are in a market where it is expensive for you to buy put, but you can write call at a fair price. Now you have decided to hedge all price risks with the call. Explain why that is possible, even though the payoff of a call only gives you upside gain.
 - We want to calculate $\frac{\partial C}{\partial S}$, which is generally denoted by δ . Looking at the Black-Scholes formula, make a guess what it will be. There is a very easy answer, but the explanation is usually wrong.

- (f) Now try to do this numerically. Calculate the call price for $S = 30$ and $S = 31.1$. Now you can approximate the derivative. Compare it to the term $N(d_1)$. Are they close to each other? (Yes, they should be because answer to (e) is $N(d_1)$)
- (g) So, how many contracts of call you have to write to hedge the risk your stock positions? What is your proceed? Express that in percentage terms of the value of your stock portfolio.
- (h) Do you need to re-balance your trades? Give two reasons why you will want to.
- 6.** Company XYZ has assets that are currently worth \$100m, and a single bond outstanding with face value \$60m and maturity of one year. The assets will only produce a single cashflow between 0 and \$100m in one year's time, and they can be liquidated for \$30m. Assume the risk free rate is zero.
- (a) Plot the Shareholders payoff next year. Does it remind you of any derivative?
- (b) Plot the Debtholders payoff next year. Does it remind you of anything?
Now assume that the cashflow generated next year can only take two values, 0 and 100, with equal likelihood.
- (c) The management (which is elected by the shareholders) is thinking about undertaking another project and financing it by issuing more debt that has the same priority as the existing bond. The new project will require an initial investment of \$30m, and its performance is directly related to the old project. Specifically, if the old project generates 0, this project will generate 0 as well, while if the old project generates \$100m, this one will generate an additional \$35m.
Given that the project has negative NPV, do you think that the firm as a whole should undertake it? Do you think that the management will want to undertake it (Consider the payoff of the shareholder if the company issues debt to finance the project)?
Can you interpret your answer?
- (d) Now the management is offered another project which needs to be financed through equity. This project will require an initial investment of \$10m, and it will generate \$35m if the old assets generate 0, but nothing if the old assets yield \$100m. In other words this project will provide some insurance.
Given that this project has positive NPV, do you think that the firm as a whole should undertake it? Do you think that the management will want to undertake it?