MIT Sloan School of Management

J. Wang
E52-456

15.407
Fall 2003

Problem Set 5: Options

Not required to hand it

1. (a) Using the put-call parity, C = P + S - PV(K) = 5 + 94 - 80*.91 = 26.2

(b) He is wrong. The put-call parity relies on the no-arbitrage argument, and if it is violated you can make risk-free profit in the market.

Since the call is overpriced, you write the call and buy a replicating portfolio, which involves buying one unit of stock, buying a put of strike 80 and borrowing PV(80) = 72.8. This will costs your 26.2, but you get 30 from writing the call. Now you have a net cashflow of 3.8, and you have a net cashflow of zero in the future.

2. (a) The intrinsic value is just max(S-K,0)

(b) The value of a call increases with maturity. Since the intrinsic value does not change, time value must increase.

(c) Case 1: Call in in the money. Notice that when strike price increase, call price goes up less than 1-for-1. However, the intrinsic value goes up 1-for-1. Therefore the time value of the option decreases.

Case 2: Call in out-of-the money. Now if strike price goes up, call value goes up but intrinsic value stays at zero. Therefore time value increases.

This example tells you that the industry definition of time value is quite artificial.

(d) If the two companies have different volatility, the one with higher volatility will have a higher option price, and hence higher time-value. The only thing that is the same is the intrinsic value.

3. (a) $p = \frac{5\% - (-5)\%}{15\% - (-5)\%} = 0.5$

(b) The option is in the money only if stock goes down in all next 3 years. Therefore, $P = \frac{(100 - 100 * 0.95^3) * p^3}{1.05^3} = 1.5401$

Constructing the replicating portfolio:

The option price evolves according to the binomial table:

t = 0	1	2	3
			0
		0	
	0		0
1.5401		0	
	3.2341		0
		6.7912	
			14.2625

At time 0, you want to short some stock so that you have a payoff of 0 if stock goes up and 3.234 if stock goes down. You can achieve that by shorting $\frac{3.2341}{115-95} = 0.1617$ units of stock, and putting \$17.7107 in the bank. This will costs you \$1.5401 today.

Similary, the replicating portfolio in the other nodes:

Time 1:Stock price goes up, unwind all transactions. Stock price goes up, short -0.3396 stock and put \$37.1925 in the bank (in total).

Time 2: unwind all transactions if stock goes up once or twice during the past two years. If stock went down twice, short -0.7131 stock and put \$77.1042 in the bank.

(c) Notice that the standard deviation of return is 10%, so we use $\sigma = 10\%$ instead of 5%. The black-scholes price is 1.7129, which is quite close.

Use the Black-Scholes formula to price the option, using a standard deviation of 5 percent per year. Contrast the result to your answer above. (Note: the standard deviation of stock return is 5 percent in this case)

(d) Just build the tree considering early exercise also. You will exercise early if stock price goes down, so the value of the put is now \$2.3801

(e) Adjust the tree manually such that the stock price goes down by an additional 20 at time 2. Note that the tree does not re-combine like it normally does (at time 3) because we are subtracting a constant amount from the nodes, not a percentage. Now the european put is worth \$8.9515 and the american put is worth \$11.6213. See the attached Excel Spreadsheet for calculations

(f) Using risk-neutral pricing gives you the "fair" price of the option, given the market prices. If the stock is correctly priced, then the option price must be such as to avoid any arbitrage.

However, it is apparent that your broker tells you that the stock is overpriced. If you agree with him, then you agree that the put is underpriced so you should buy the put. However, this is a subjective judgement, and you are not making risk-free profits.

4. Use the Put-Call parity, C - P = S - PV(K). However, if K = F, then we know that PV(F) = S. Therefore C - P = 0.

5. (a) The futures usually trades in larger amounts. It is impossible to use it to hedge a few stocks that you hold. Also margin requirement is inconvenient for small investor. But with options you don't have such problems.

(b) Buy 1000 puts.

(c) Each put costs \$1.3392, so you need to pay \$1339.2 to hedge for 1 year.

(d) Although the call have only upside, it is always sensitive to both up- and down-movement in stock price. Therefore, given you can re-balance frequently, you can still use the call to hedge the downside risk of a stock.

(e) $\delta = N(d_1)$ in the Black-Scholes formula. But it is not because simply $C = S * N(d_1) - K * B * N(d_2)$, and you take a simple derivative. The term d_1 and d_2 also have the term S in it, so it is a great coincidence that the two matches up.

(f) When S = 31.1, C = 3.8722. When S = 31, C = 3.8023. Therefore $\delta \approx \frac{3.8722-3.8023}{0.1} = 0.6992$, which is very close to $N(d_1) = 0.6964$

(g) You will need to write 1000/0.6964 = 1436 contracts. Your proceed is \$5460.25, which is 18.2% of your stock value. However you need to adjust your portfolio throughout the life of the option, which will costs you to use up your proceeds.

(h) You will need to re-balance your trades. Suppose the stock price stay constant. Then as time goes by, the stock is worth the same, but the call is more sensitive to stock price. So you will want to buy back some calls. On the other hand, if the call moves more deeply into money, the call is also more sensitive to stock price. So you will want to re-balance as often as possible.

6. (a) It is the same as long a call option plus cash.

(b) It is the same as short a put option plus cash.

Now assume that the cashflow generated next year can only take two values, 0 and 100, with equal likelihood.

(c) We need to determine the risk-neutral probabilities. Assume the risk-free rate is zero for simplicity. Notice that $p * 100 + (1 - p) * 0 + 30 = 100$, so p = 0.7.

So the value of the project is 0.7*35 = 24.5, so it has negative NPV.

If the management rejects the project, the stock holders get nothing if the asset produces 0 and get 70 if the asset generates 100. Therefore the value of the stock is 49.

Now if the manage takes the project, the stock holders still get 0 if asset produces 0 but get 75 if the asset generates cash. Therefore the value of stock is increased to 52.5. If the management cares only about stockholders but not debtholders, they will undertake the new project, hurting the debtholders.

This illustrates the agency problem that the shareholders is inclined to make the company more risky, even if it may not be efficient. This makes the debt more risky, thus have a lower value and the shareholders benefit from that.

(d) Assume the asset pay 35 in the down state (because 30 won't give you a positive NPV)

The value of the project = 0.3*35 = 10.5, so NPV is positive.

Now repeat the exercise as above. The shareholders still get 0 if the company produces low cashflow but now only 60 if the company produces high cashflow. Thus the value of stock drops to 42, but the risk of debt has greatly decreased.