MIT Sloan School of Management

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Problem Set 6: Interest Rates

Due: November 14, 2003

1. Sally Jameson: Valuing stock options in a compensation package

Read the HBS case, "Sally Jameson: Valuing Stock Options in a Compensation Package", that is included in the Reading Package. Then, please help Sally determine the value of the option that she has been offered. 1-2 page write up, calculations not included, should be sufficient. Remember to demonstrate your calculations.

2. (BKM) Answer the followings:

(a) Briefly explain why bonds of different maturities have different yields in terms of the (1) expectations, and (2) liquidity.

(b) Briefly describe the implications of each of the two hypotheses when the yield curve is (1) upward sloping, and (2) downward sloping.

(c) Under the liquidity preference theory, if inflation is expected to be falling over the next few years, long-term interest rates will be higher than short-term rates. True/false/uncertain? Why?

3. Here is the spot yield curve derived from the Expectation Hypothesis:

Maturity	1	2	3	4	5	6	7
Spot Rate	2.0%	2.2%	2.5%	2.8%	3.2%	3.5%	3.6%

What would be the spot yield curve under the Liquidity Preference Hypothesis? Assume the risk premium $\pi_t = 0.1\%$*t.

4. The representative investor has an endowment of e_0 today and e_1 tomorrow. He maximizes the following objective function:

$$\max \quad u(c_0) + \rho u(c_1)$$

such that

$$\begin{aligned} c_0 &= e_0 - b \\ c_1 &= e_1 + (1+r)b \end{aligned}$$

where b is the bond holdings. The investor's utility function is defined by

$$u(c) = -e^{-\alpha c}$$

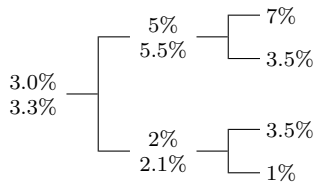
where α is some positive constant.

(a) Solve for the equilibrium interest rate r?

(b) How does r changes when the representative investor's next day endowment, e_1, increases?

(c) How does r changes with investor time-patience ρ?

5. Consider the following binomial-tree interest rate model for $t = 0, 1, 2$



In each node, the first number is the one-year spot rate and the second number is the two-year spot rate at time t. The two-year rate is not available at t = 2.

(a) Construct a tree that shows the evolution of one-year and two-year discount bond prices.

(b) In time 0, form a portfolio of one-year and two-year discount bonds that replicates the one-period payoff of the three-year discount bond. What is the price of the three-year discount bond in time 0? What is the three-year spot rate? (Hint: Note that an N-year discount bond will become (N-1)-year discount bon after a year. The method to replicate the payoff of the three-year discount bond is identical to the method that we use to price option. The analog of stock price process is the price process of the two-year discount bond. The analog of the option payoff is the price of the three-year bond one year later.)

(c) Someone issues a three-year 10% coupon bond with par value 100 at time 0. Construct a binomial tree to show how the ex-coupon price of such bond changes through time.

(d) Now, someone issues a similar bond except that the bond is callable at \$110 in time 1 (i.e. the seller of the bond can buy back the bond at \$110 at time 1). What is the value of the embedded option in the callable bond?