

MIT SLOAN SCHOOL OF MANAGEMENT

J. Wang
E52-456

15.407
Fall 2003

Problem Set 7: Portfolio Choice

Due: November 25, 2003

1. (BKM) Which statement(s) about portfolio diversification is correct?
 - (a) Proper diversification can reduce or eliminate systematic risk.
 - (b) Diversification reduces the portfolio's expected return because it reduces the portfolio's total risk.
 - (c) As more securities are added to a portfolio, total risk would typically be expected to fall at a decreasing rate.
 - (d) The risk-reducing benefits of diversification do not occur meaningfully until at least 30 individual securities are included in the portfolio.

2. Consider the following scenario: There are an infinite number of stocks in the economy, and each of them has a annual volatility (standard deviation) of 30%. The covariance between any two stocks are 0.01. The risk-free rate is 5%.
 - (a) What is the magnitude of the systematic risk in the stock market?
 - (b) Assuming perfect market (no trading cost, short sell allowed, etc) is it still possible for some stocks in the market to have different expected returns?

3. There are only two securities (A and B, no risk free asset) in the market. The correlation between returns of A and B is -0.4. Expected returns and standard deviations are as follows:

Security	Expected Return	standard Deviation
Stock A	20%	20%
Stock B	15%	25%

- (a) Would anyone be interested in investing in B? If so, why?
- (b) What is the expected return and standard deviation of a portfolio that invests 60% in A and 40% in B?
- (c) Suppose your client wants to form a portfolio out of these securities with the lowest standard deviation. Solve for the portfolio weights analytically. What is the expected return and standard deviation of your portfolio?
- (d) Now introduce a riskless asset, and the risk free rate is 4%. Now, fixing the standard deviation at a constant level, solve for the portfolio (of the 3 assets) that gives you maximal return. This gives you the tangent portfolio.

- (e) Now, using the results in (d), what is the optimal portfolio if your client wants a return of (i) 10%, (ii) 20%? What is the ratio between wealth invested in A and B in each case?

4. Portfolio Choice Case - “Beta Management Company”

Read the HBS Case, “Beta Management Company”, that is included in the Reading Package. Then, answer the following questions.

- (a) Calculate the variability (standard deviation) of the stock returns of California REIT and Brown Group during the last 2 years. How variable are they compared with Vanguard Index 500 Trust. Which stock appears to be riskier?
- (b) Suppose that Beta’s portfolio position has been 99% of equity funds invested in the index fund (Vanguard) and 1% in individual stock. Calculate the variability of this using each stock in turn. How does each stock affect the variability of the equity investment, and which stock is riskiest? Compare this to your answer to the previous question, and explain your response.
- (c) Perform a regression of each stock’s monthly return on the Index returns to compute the “beta” for each stock. How does this relate to the situation described in the question above.
- (d) How might the expected return for each stock relate to its riskiness; that is, which stock should have a higher expected return?

5. Portfolio choice in real life: Application and problems.

Download the data file from the class website (PT_data.xls). It contains the past 15 years of prices of 8 big stocks.

- (a) Calculate the average and standard deviation of the return on each stock. Remember to transform prices to returns first.
- (b) Using only data from 1988-1998, create the mean-variance frontier with short selling not allowed. (Be careful because some returns are not achievable, so you need to program accordingly)
- (c) Now do it again, but also allows for short-selling. Compare and contrast the result in here with your results in (b)
- (d) Now, apply your “efficient portfolio” you got in (c), and calculate the mean and standard deviation of your returns if you invested in this portfolio from 1999-2003. On a graph, plot the mean-variance frontier and the realized mean-variance frontier. What kind of return and standard deviation did you get? Is it similar to what you have expected?
- (e) (Extra credit) Give a few reasons to explain your findings in (d)