MIT Sloan School of Management

J. Wang
E52-456

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# Problem Set 8: Portfolio Choice

Solution

**1.** (a) False. Stocks with zero beta offers the risk free rate.

(b) False. Investors require a risk premium only for bearing systematic risk (undiversifiable or market) risk.

(c) False. Since beta is linear, you can construct a portfolio with beta of 0.75 by buying 75% market portfolio (which has beta 1) and 25% in Tbills.

**2.** (BKM-revised) In 1999 the rate of return on short-term government securities (perceived to be risk-free) was about 5%. Suppose the expected rate of return required by the market for a portfolio with a beta of 1 is 13%. According to the CAPM:

(a) The market portfolio has the same expected return as a portfolio of beta of 1, that is, 13%.

(b) That will be the same as the risk-free asset, 5%.

(c) Expected return is 13%(include dividend), and that is higher than normal for a stock with beta of only 0.5. Therefore the stock is underpriced.

**3.** (a) The variance of the portfolio is

$$\begin{aligned}\sigma_p^2 &= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\sigma_{12} \\ &= (.75^2)(.16) + (.25^2)(.09) + 2(.75)(.25)(.02) \\ &= .103.\end{aligned}$$

(b) We can calculate $\beta$ by

$$\begin{aligned}\beta_1 &= \frac{\sigma_{1m}}{\sigma_m^2} = \frac{.064}{.04} = 1.6 \\ \beta_2 &= \frac{\sigma_{2m}}{\sigma_m^2} = \frac{.032}{.04} = .8 \\ \beta_p &= (.75)\beta_1 + (.25)\beta_2 = 1.4.\end{aligned}$$

(c) We find $R^2$ by

$$\begin{aligned}R_1^2 &= \frac{\beta_1^2\sigma_m^2}{\sigma_1^2} = \frac{(1.6^2)(.04)}{.16} = .64 \\ R_2^2 &= \frac{(.8^2)(.04)}{.09} = .284 \\ R_p^2 &= \frac{(1.4^2)(.04)}{.103} = .760.\end{aligned}$$

(d) According to the CAPM, the expected return of security $i$ is calculated according to

$$\begin{aligned}
\bar{r}_i &= \bar{r}_f + \beta_i(\bar{r}_m - \bar{r}_f) \\
\bar{r}_1 &= .04 + 1.6(.12 - .04) = .168 \\
\bar{r}_2 &= .04 + .8(.12 - .04) = .104 \\
\bar{r}_p &= .04 + 1.4(.12 - .04) = .152.
\end{aligned}$$

(e) You will invest only in the market portfolio (and the riskless asset), but at the same time a fraction of the market portfolio will be stock 1 and stock 2.

**4.** (a) True. By definition, the factors represent macroeconomic risks which cannot be eliminated by diversification.

(b) False. The APT does not specify the factors.

(c) False. The APT does not specify the sign of the factor loadings.

(d) True. Different researchers have proposed and empirically investigated different factors, but there is no widely accepted theory as to what these factors should be.

(e) True. To be useful, we must be able to estimate the relevant parameters; if this is impossible, for whatever reason, the model itself will be of theoretical interest only.

**5.** Let $r_x$ be the expected risk premium of investment $X$ etc., and let $x_x$ be the portfolio weight of $X$, etc.

(a)

$$\begin{aligned}
r_x &= 1.75\,(.04) + .25\,(.08) = .09 \\
r_y &= -1.0\,(.04) + 20\,(.08) = .12 \\
r_z &= 2.0\,(.04) + 1.0\,(.08) = .16.
\end{aligned}$$

(b) This portfolio has the following portfolio weights:

$$\begin{aligned}
x_x &= \frac{200}{(200 + 50 - 150)} = 2.0 \\
x_y &= \frac{50}{(200 + 50 - 150)} = .5 \\
x_z &= \frac{-150}{(200 + 50 - 150)} = -1.5.
\end{aligned}$$

The sensitivities of this portfolio to the factors are:

$$\begin{aligned}
\text{Factor 1} &: \quad 2.0\,(1.75) + .5\,(-1.0) - 1.5\,(2.0) = 0 \\
\text{Factor 2} &: \quad 2.0\,(.25) + .5\,(2.0) - 1.5\,(1.0) = 0.
\end{aligned}$$

Because the sensitivities are both zero, the expected risk premium is zero.