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15.407
Fall 2003

Midterm Solution

1. (a) False. If the cashflows are of different risk nature and timing, then you need to use the corresponding discount rate for each cashflow.

(b) False. Normally, the value of a bond decrease when interest rate rise because the present value of future payments are decreased. However, there is an additional effect of the reverse floater that the promised payment is decreased, pushing the price even further down. Therefore, the modified duration on a reverse floating must be higher than the its straight bond counterpart.

(c) False. If an investor is short term, i.e. he plans to sell the stock after a few dividend payment, then when he sells the stock, the price he could sell is still the discounted value of all future dividends. Therefore, the value of the stock to him, which is the discounted value of all the dividends he gets before he sells the stock plus the resell value of the stock, is the same as that is predicted by the DDM.

(d) False. A growth company has investment opportunities with expected return higher than the required rate of return. However, earnings may not be growing. For example, the firm could be investing heavily, leading to lower current earnings.

(e) False. If the dividend yield exceeds the corresponding spot riskless rate, then the futures price will drop below the spot price.

(f) Uncertain. For an american call, this must hold because by holding a call of longer maturity, the worst you can do is just exercise it early.

For european option, if a call of shorter maturity has value higher than one of longer maturity, then that must mean that being able exercise earlier has positive value. However, we know that you will only want to exercise a call earlier when the stock pays dividend. Therefore, the only time that the call value does not increase with maturity is when we know that if we increase the life of the option, we will get hit by a big dividend payment.

2. (a) For the scam to be profitable, the interest you gain from using this strategy must exceed the withdrawal fee. Therefore, your cash advance limit (CA)is given by:

$10 \leq CA * [(1 + .25\%)^3 - 1]$

$CA \geq \$1330.01$

3. (a) $P = \sum_{t=1}^{4} \frac{\$100*1.05^t}{(1+5\%)^t} = \$400.$

(b) Since the PV of each payment are equal (to \$100), the duration is just (1 + 2 + 3 + 4)/4 = 2.5

(c) Suppose Yankee Inc. wants to invest x of its proceed into the 12-month T-bill and (1-x) into the 4-year STRIPS.

Then the duration of the position will be x*D(T-bill) + (1-x)*D(STRIPS) = x* + (1-x)*4 = 4 - 3x.

To avoid all interest rate risk, we want -10m * 2.5 + 10m*(4-3x) = 0. This means that x = 0.5, so Yankee Inc. should buy $5m worth of 12-month T-bill and $5m worth of STRIPS.

(Note, as the yield curve is flat at 5%, YTM of all bonds are 5%, so hedging with duration is equivalent to hedging with modified-duration, and both are acceptable)

4. (a) $D_1 = \$5 * (1 - 40\%) = \$3, g = ROE * b = 4.8\%$
So $S = \frac{D_1}{r-g} = 71.4286, P/E = 14.2857$

(b) That depends on the ROE on the new investment projects.

If the new project still has an ROE higher than 9%, then P/E will increase. The new project give higher return than what the market has requested.

If the new project has ROE = 9%, then the value of the company will not change.

If the new project has ROE less than 9%, then the new investment delivers return lower than what the market has asked for, so it will decrease the value of the company.

(c) It is unreasonable to assume that the firm can forever invest at 12% return, because the the company will run out of good investment projects. Therefore, let's assume that the ROE of the new project falls as you increase the plowback ratio. Therefore, to maximize current stock price, I will advice the company to increase plowback, as long as it is able to invest at an ROE higher than 9%. Once those project runs out, the company should stock investing in new projects and pay the remaining earning out as dividend.

5. (a) Spot price = $\frac{\$312}{1.04} = \$300$

(b) $r_2 = \sqrt{\frac{327.6}{300}} - 1 = 4.5\%$
Similarly, $r_3 = 5\%, r_4 = 5.49\%$

(c) $f_3 = \frac{(1+r_4)^4}{(1+r_3)^3} - 1 = 7.00\%$

6. (a) We first calculate the risk-neutral probability, q. $q = \frac{R-d}{u-d} = \frac{1.05-.9}{1.15-.9} = 0.6$

Next, we find the payoff of the american option at time 2. Let the price of the american at each node be:

t	0	1	2
			p_{uu}
		p_u	
	p		p_{ud}
		p_d	
			p_{dd}

Then $p_{uu} = max(100 - 100 * 1.15^2, 0) = 0, p_{ud} = max(100 - 100 * 1.15 * 0.9, 0) = 0, pdd = 100 - 100 * 0.9^2 = 19$

Therefore, at time 1, if stock price has gone up to \$115, exercising give you zero profit, whereas waiting give you $(q * p_{uu} + (1 - q) * p_{ud})/(1 + r)$, which is also zero, so $p_u = 0$

If stock price went down to \$90, then exercising gives you \$10, and waiting gives you $(1 - q) * 19/(1 + 5\%) = 7.2381$. Therefore exercising gives you a higher value and $p_d = 10$

Back to time 0, you will not exercise the option that is at-the-money, so $p = (1 - q) * p_d/(1 + 5\%) = 3.8095$.

(b) The replicating strategy at time 0 gives you 0 if price goes up, and 10 if stock price goes down at time 1. Assume you hold invest $\$\alpha$ in the riskless asset and β units of stock.

Then $1.05 * \alpha + 115 * \beta = 0$ and $1.05 * \alpha + 90 * \beta = 10$, solving this yields $\alpha = 43.8095, \beta = -0.4$, the cost of this today is 43.8095 - 0.4*100 = 3.8095 = p

At time 1, you always liquidate your position, because (i) if the stock price goes up, your net position is zero and the option will never be exercised, and (ii) if stock price goes down, you will exercise your option, so you should liquidate all of your replicating portfolio. You don't hold any stock into time 2 whether stock prices went up or down.