15.407 Recitation

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Things to cover today:

Fixed Income Concepts:

1. Bonds
2. Spot rates, Forward rates and YTM
3. Duration and Convexity
4. Market conventions

Fixed Income: Very roughly, things that have a promised payments, and derivatives drawn on them:
Examples: Bonds, Swaps, Credit derviatives
Today we will focus on (riskless) bonds.

How to price a bond?
If payments are riskless, then we can just use the PV formula.

But the difference between today and last week is that we do not yield curve (interest rates) is constant anymore.

Characteristics of bonds:
(A)Distribution of cashflow:
(i) Discount bond: Bonds that pays you a fixed amount at maturity
(ii) Coupon bond: Bonds that pays you in many periods (coupon), and the principle + coupon at the end

(B) Rate of payment:
(i) Fixed (Nominal): Have a constant nominal interest rate.
(ii) Floating: Interest rate varies each period.
(iii) TIPS: Constant REAL interest rate.

(C) Maturity
Bills: less than 1 year maturity
Notes: Between 1 to 10 years
Bond: 10 years or more

(D) Level of risk

Interest rates and yields: Spot rate

The current interest rate for a fixed maturity.

Usually denote by r_t the t-year spot rate.

Example: Consider a bond with n-year maturity, paying a coupon of C each year with principle of P.

Price of bond $= B = \sum_{i=1}^{n-1} \frac{C}{(1+r_i)^i} + \frac{C+P}{(1+r_n)^n}$

Forward rate

The rate in the future that you can lock in today.

Usually denote f_t the t-year forward of 1-year rate.

What is the price of the bond above?

$B = \frac{C}{1+f_1} + \frac{C}{(1+f_1)(1+f_2)} + \ldots + \frac{C+P}{(1+f_1)(1+f_2)\ldots(1+f_n)}$

We will use the forward rate extensively when we go to forwards/future and option pricing.

Question: How can we derive forward rates from spot rates, and vice versa?

Yield-to-Maturity

A CONSTANT rate of discount that makes the PV of the bond same as its cost (same as IRR)

denote by y - specific to each bond!!

$$B = \sum_{i=1}^{n-1} \frac{C}{(1+y)^i} + \frac{C+P}{(1+y)^n}$$

Usually we have to solve for y numerically.

Duration and Convexity

Basic idea: Bond prices changes as interest rate changes.

Duration is the absolute change in bond value as the yield changes. It is the first derivative of bond price over yield.

We can calculate the duration by the formula: $D = \sum_{i=1}^{n} \frac{PV_i * i}{B}$, where PV_i denote the present value of the i-th payment of the bond.

if we divide duration by $1+y$, we get the Modified Duration, (MD) which measures the percentage change in price as y changes.

Duration Hedging

To hedge against duration risk of a portfolio, we just need any instrument with MD not equal to zero. Then we can take opposite position such that:

Change in value in portfolio + Change in value in hedge = 0

Question: Although anything with non-zero duration could be used as a hedge, are there better instruments than others?

Answer: Yes, since duration changes as yield changes.

Factors that determine sensitivity of duration:

- Yield
- Maturity
- Coupon rate

Other than this, there is also a cost factor. Hedge that requires you to deposit/pay a large sum of cash is usually not desirable.

Convexity:
Convexity the second derivative of price over yield. It measures how sensitive the duration to yield changes.

Question: So is a positive convexity a good thing or bad thing?

Remember that duration only hedges against the linear change in price. If you expect a big movement in rate, then you may want to hedge convexity too.

Market Convention

Clean and Dirty Price:

Using our formula, we are always calculating the dirty price of the bond.

Assuming the interest should be paid continuously, however, we calculate the accrued interest, subtract it from the dirty price, and we get the clean price, which is being quoted on the Wall Street Journal.

Assume it is x days since the last coupon payment and the number day between the last and next coupon payment is y days, then for a bond that pays k coupons per year

Accrued Interest $= \frac{x}{y} * \frac{C}{k}$

You will use this in the problem set.