15.407 Recitation

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MIT Sloan School of Management

Things to cover today:

Forwards and Futures:

1. Questions about Equity pricing?

2. Forwards and Futures - Market Convention

3. Pricing

4. Hedging

Definitions:

Forwards: A commitment to purchase at a future date a given amount of a commodity or an asset at a price agreed on today.

Futures: An exchange-traded, standardized, forward-like contract that is marked to the market daily. This contract can be used to establish a long (or short) position in the underlying asset.

Example of a future contracts:

30 Year T-bond future on CBOT.

Contract Size: One U.S. Treasury bond having a face value at maturity of $100,000 or multiple thereof.

Deliverable Grades: U.S. Treasury bonds that, if callable, are not callable for at least 15 years from the first day of the delivery month or, if not callable, have a maturity of at least 15 years from the first day of the delivery month. The invoice price equals the futures settlement price times a conversion factor plus accrued interest. The conversion factor is the price of the

delivered bond ($1 par value) to yield 6 percent.
Tick Size: 1/32 of a point ($31.25/contract); par is on the basis of 100 points
Price Quote: Points ($1,000) and thirty-seconds of a point; for example, 80-16 equals 80 16/32
Contract Months: Mar, Jun, Sep, Dec
Last Trading Day: Seventh business day preceding the last business day of the delivery month.
Last Delivery Day: Last business day of the delivery month.
Trading Hours: Open Auction: 7:20 am - 2:00 pm, Chicago time, Mon - Fr
Electronic: 8:00 pm - 4:00 pm CT Sun - Fri
Trading in expiring contracts closes at noon CT on the last trading day
Ticker Symbols: Open Auction: US, Electronic: ZB
Daily Price Limit: None

Differences between Forwards and Futures
(I) Futures contracts are standardized, exchange traded:

• Contracts are standardized: Very few (usually just one) contact offered for each good for a limited numbers of maturity dates.

• Non-trivial bid-ask spread

• Open interest is generally higher for shorter maturity, reflecting higher volume

(II) The contracts are guaranteed by clearing house:

Suppose you buy 100 futures on a 10 Year Strip (Face value of $10 million), and interest rates went up by 25 basis point (0.25%), what is your loss?

Roughly, 10 * .25 = 2.5% of the market price of market price of the STRIP (about $6.3m), so its a loss of $165K. Therefore you really don't want to honor the contract.

Default risk in forward should not be neglected.

(III) Marked-to-Market and Margin requirement:

In order to prevent large loss for the clearing house, trades are marked-to-market daily. You are required to deposit an initial margin (relative to the size of your trades, say 20%) into the clearing house.

Each day, the clearing house calculate the change in future prices from the previous day, and immediately charge you for any gain/loss.

If your margin falls below a certain level (10%), you have to deposit addition funds into the clearing house back to the initial margin (20%).

Nowadays, many dealers apply the similar margin requirement for forwards.

How do we price forwards?

If we abstract from liquidity and limited credit risk (as discussed above), we can use no arbitrage!

Example 1: Assume the the 6-month yield is 2% and the 1-year yield is 3% (both semi-annual). You want to buy a 6-month forward on a 1-year zero-coupon T-note, face value \$100. What is the forward price?

Answer: Remember that no cash changes hand today.

Price of note today $= \frac{100}{1.015^2} = \$97.066$

So if we borrow \$97.066 for six-month and buy the note now, we replicate the forward. We need to pay back $97.066 * 1.01 = \$98.037$ in 6-month, so that must be the forward price.

So we have $F_t = P_0(1 + r_t)^t$, where r_t is the annualized spot rate.

Convenience yield:
The above equation does not always hold on the exchange. Why?
For some assets, there is cost/benefit associated with holding the physical asset.
Crude Oil: Costly to store (c% per year), Index: Dividend paid - almost continuously (y% per year).
Define y = y - c, then $F_t = P_0(1 + r_t + y)^t$
Example 2: The 1-year forward price on Gold is 380.00 and Crude Oil is 25.35. The spot prices of them are 375.25 and 29.11 respectively. Current 1-year STRIP price is $98.22. What are the convenience yield charged by the market for each commodity?

Answer: The one year spot rate is 1.812%. Therefore, $380 = 375.25 * (1 + 1.812\% + y)$, so y = - 0.5464%. Similarly the net convenience yield = 13.0201% for oil.

Hedging

Suppose you manage 123,456 oz of gold for your client, and you are worried that gold price may fall in 1 year, but you cannot sell them.
You can short the same amount in the forwards market for 11 months. Now you are completely immune to the gold risk for 11 months.
Is there any problem, though?
(i) You can use a forward, but it may be hard to find someone to take on all the risk, considering the size of your position. Trading with futures may be easier, but contracts are standardized and may not fit your need perfectly.
(ii) If your client decides to sell half of the gold after 6-month, you may not be well protected.
(iii) Even if your client does not change his mind, gold prices can fluctuate a lot in the next 11 months. If you don't have enough liquid asset to cover your short position in the futures, you may run into margin calls.

So, is there a better way to hedge?

Statistical idea: Minimize some measure of your expected loss, which is equivalent to minimizing the expected deviation of your hedging portfolio to your actual portfolio.

Minimal Variance Hedging:
Regress historical changes in stock price on the historical changes in future prices. The coefficient β you get is tells you this: If I hedge one unit of stock with β units of futures, I minimize the standard deviation of my portfolio.
So β is the optimal hedge ratio! Analytically, $\beta = \frac{Cov(\Delta S_t, \Delta H_t)}{Var(\Delta H_T)}$
Assumption used here: the factors affecting the relation between changes in stock prices and future prices are the same. Same as what people usually mean when they say (Covariance) Stationary.
Note: There are possible ways to hedge more perfectly, but this model requires the minimal assumption and is very easy to implement.