15.407 Recitation

October 23, 2003

MIT Sloan School of Management

Things to cover today:

Options:

1. Price bounds for options
2. American option and early exercise
3. Factors affecting price of option
4. Ways to price options
5. Risk Neutral Pricing

Price bounds:
There are upper bounds and lower bounds to the price of all options.

Let B be the price of a discount bond with maturity same as futures.
For calls, $S - KB \leq C \leq c \leq S$
For puts, $K \geq p \geq P \geq \frac{K}{B} - S$
The bounds are usually not tight, but figuring out how to get the bounds is a very good test of your understanding to no-arbitrage.

Put-Call Parity:
This always hold in equality for European calls and puts on stocks that don't pay dividend.
$C - P = S - KB$, where C and P have same strike (K) and maturity (same as B)
If LHS $\geq$ RHS, then you just buy the RHS portfolio and short the LHS portfolio.
To buy the RHS portfolio, you buy one unit of stock, and borrow KB. You will need to pay S-KB as a cost.

Then you write a call and buy a put, this will give you a payment larger than your cost of buying the RHS portfolio. However, you will always have zero payoff at maturity.

Question: What happen if the stock pays dividend?

Question 2: What about American options? For American option on stocks that don't pay dividend,

$S - K \leq c - p \leq S - KB$

American Option and Early Exercise:
American Option allow you to exercise any time before maturity. However, would you ever want to?

Consider owning an american call on a stock that is very deep in the money. You are considering if you should exercise it, or wait until it matures in 6 months. The stock does not pay dividend.
It is true that the option may lose value if you don't exercise it now. However, you can protect yourself against the lost by shorting the stock.

Combining a call and a short on stock, it is the same as having a put. Therefore, instead of paying the strike K immediately, you don't pay K until the option expires.

However, if the stock pays dividend, then you may want to exercise early. Still, there are only certain points in the life of the option you will consider to exercise it.

Puts: Now it is less clear. To protect yourself against potential loss, you will have to buy the stock, which costs you to pay more upfront. Therefore, for any stock you will consider exercising an option early.

You will see this more clearly when we go to binomial pricing.

Factors that affect the price of an option:
What happen to the prices of calls and puts when these factors of the underlying goes up?

Factor	Call	Put
Price	Increase	Decrease
Strike		
Time to maturity		
Interest rate		
Volatility		
Dividend		

Factors that affect the price of an option:

There are, in general, 3 ways to do pricing:

(I) Close form solution:(Black-Scholes)

(II) Numerical Approximation:(Binomial Tree)

(III) Simulation

Black Scholes Formula:

The price of a european call is:

$S * N(d_1) - K * B * N(d_2)$

$d_1 = \frac{ln(\frac{S}{K}) + (r + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}}, d_2 = d_1 - \sigma\sqrt{T}$

The price of a european put is:

$K * B * N(-d_2) - S * N(-d_1)$

Assumption of Black-Scholes model:

- Continuously compounded returns are normal
- Stock does not pay dividend
- Interest rate is constant
- Options are European
- Volatility is constant

Although these assumptions may not all hold, the Black-Scholes formula is very useful we just have to be careful when these imperfections can lead to a big price difference. For example, using the Black-Scholes formula on a long-dated American put is very dangerous.

There are also extensions to Black-Scholes formula to take care of stochastic volatility and approximations for american features.

Binomial Tree and Risk Neutral Pricing:
Idea: assume stock price can only go up or go down, also assume that we can linearly price the asset. Any objections?

This looks quite awkward, but it is theoretically justified.
What we are assuming:
(i) There is no arbitrage
(ii) There is no market fricition

What we are **NOT** assuming:
(i) Everyone is risk-neutral
(ii) We can calculate prices of everything using **TRUE PROBABILITY**

Let's look at one simple example:

Stock price one year from now could only be 25 or 5. An asset that gives you 1 only if the stock price is 25 tomorrow costs 0.2, whereas an asset that gives you 1 if the stock price is 5 costs 0.6. What is the price of the stock?

Using no-arbitrage, you can see the price is 25*0.2 + 5*0.6 = 8
But consider the interest rate being
$\frac{1}{0.2+0.6} - 1 = 25\%$, $p_1 = \frac{0.2}{0.8} = 0.25$, $p_2 = 0.75$, the discounted expected payoff of the stock is also $\frac{25*.25+5*.75}{1.25} = 8$

Therefore linear pricing is equivalent to using no-arbitrage.

In binomial tree, we assume the price can only go up by a ratio of u or down by a ratio of 1/u = d.
Then $p_1 = \frac{R-d}{u-d}$, $p_2 = fracu - Ru - d$
Remember we can increase the frequency of the steps. Under the black-scholes assumptions, the binomial price converges to the black-scholes price.

See attached excel spreadsheet for an illustration of early exercise feature.