15.407 Recitation

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Interest rates and risks:

Things to cover today:

1. Theory of interest rates

2. Risk and its measurements

Characteristics of interest rates:

- (Nominal) positive
- Upward sloping in general
- Low volatility
- Mean Reverting
- Big market, can be very risky leverage is high!
- Three factors explain almost all of variations in interest rates: level, slope and curvature.

So, how can we model interest rates?

Models of interest rates:

- Endowment model
- Expectation/Liquidity Hypothesis
- Models of the term structure

Model of endowment

This is the usual utility maximization problem: Max U(c) given endowment. See class note for an example.

You can build in more assumptions about the model. However, these models do a very poor job explaining interest rates.

Expectation hypothesis:
$f_{t,k} = E[r_{t,k}]$. t denotes the starting date of the rate and k denotes the duration of the interest rate.
i.e. $f_{1,1}$ is the one year forward of the one year interest rate, which is known now. $r_{1,1}$ is the one-year spot rate that takes place one year from now, which will not be known until one year later.

Expectation hypothesis not popular anymore.

Liquidity hypothesis:
$f_{t,k} = E[r_{t,k}] + \pi_t$, a premium is added if you promise to take on interest rates in the future. You can think about it as a compensation of taking interest rate risks.
But liquidity hypothesis is just a very reduced-form model.

Models of term structure:
These models only model how interest rate changes. Contrary to endowment model, they do not tell anything about how interest rate is determined in equilibrium.

But they are very useful in industry! You care a lot more about matching the yield curve then to understand the underlying economic reasoning.

Examples of term-structure models: (Roughly in order of complication)

- Vasecik
- Ho-Lee
- Cox-Ingersoll-Ross
- Heath-Jarrow-Morton

These models are much beyond the scope of 15.407....
Reference: Hull (Option, Futures and other Derviatives). Sundaresan (Fixed Income Market and their Derivatives)

Risk:

Question: What is risk?

Confine ourselves to financial risk. Then it is anything that affect your wealth.

To summarize, we usually only use average return and standard deviation of return. This works if either one of the following assumptions holds:

- Returns are normally distributed.
- Investor only care about the mean and standard deviation of returns.

Are these good assumptions? Let's look at stocks:

Characteristics of stock returns:

- Mean return roughly 12%, volatility 20% per year.
- Fat tails
- Negatively skewed, excess kurtosis
- Not normal!

However, we still continue to operate on the assumption that stock returns are normal, because of its tractability.

- Cross section: Contemporary returns of stocks are correlated
- Time series: For the same stock, returns over time does not show strong auto-correlation.

Calculations of portfolio returns:

Suppose you have a portfolio of n stocks, and weights $\{w_1, w_2, ...w_n\}$ on each stock.

For stock i, expected return is r_i, variance (squared volatility) is σ_i^2, and covariance with stock j be σ_{ij}.

Denote the mean and volatility of the portfolio return be r_p and σ_p, then

$$\begin{aligned} r_p &= \sum_{i=1}^{n} w_i r_i \\ \sigma_p^2 &= Cov(\sum_{i=1}^{n} w_i r_i, \sum_{j=1}^{n} w_j r_j) \\ &= \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j Cov(r_i, r_j) \\ &= \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i \neq j} w_i w_j \sigma_{ij} \end{aligned}$$

This is important when we come to portfolio choice.

Regression analysis:

You will use this later on, so this is a walk-through for the basics of a regression:

Linear regression: You assume the following relation:

$$y = \alpha + x_1\beta_1 + x_2\beta_2 + ... + x_m\beta_m$$

Therefore, suppose we have n observations for y $(y_1, ..., y_m)$ and same for each x. That means for each observation we have:

$$y_k = \alpha + x_{1k}\beta_1 + x_{2k}\beta_2 + ... + x_{mk}\beta_m + \epsilon_k$$

We have an error term (ϵ) because the relation may not hold exactly in the data.

(Sometimes people prefer to use β_0 as the intercept rather than α.)
Linear regression fits (constant) values for all β's (calls $\widehat{\beta}$,we put hat on anything we estimate) such that the sum of squared errors ($\sum_{j=1}^{n} \widehat{\epsilon}_j^2$) is minimized. You can use a number of statistical software to do it. In Excel, you can either use the Analysis Toolbox (nice output, but is a black box) or LINEST command (Could be confusing).

Key statistics:
α and β's: These are the best estimates you can get.
Standard error: This is how volatile/reliable your estimate is. It is always better to have smaller standard error, but you can only achieve that if you have additional data.

R^2: This tells you how much of the variation in the y's are measured by your x's. This is important in predictive regression, because it tells you how well you can predict things in the future. Otherwise, you may not care as much about R^2 because: (1) You can always increase R^2 if you increase the number of variables in x and (2) For different regressions, R^2 is usually not comparable.

Hypothesis testing:
Suppose you ran a regression and get $\widehat{\beta} = 1$. And you expected $\beta = 0.5$. What does this mean?

You want to find out how likely a regression will yield $\beta = 1$ if the true β is 0.5. For this we use a t-test.

Suppose your regression gives you $\widehat{\beta}$, and you want to test $\widehat{\beta} = \beta$. Then you calculate $t-stat = \frac{\widehat{\beta}-\beta}{SE_{\widehat{\beta}}}$. A rule of thumb is that if the absolute value of t-stat is larger than 2 (1.96), then we say $\widehat{\beta}$ is significantly different from β (with 95% confidence). What this really means is that, suppose β is really 0.5, there is a less than 5% chance that we actually end up with the data we get for the regression. This is the two-sided t-test.

Caution! If the t-stat is not large enough, we do not reject the hypothesis. That does not mean that we ACCEPT the hypothesis, we just don't know it is true or not.

In general, we can only reject but not accept a hypothesis.