MIT Sloan School of Management

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E52-456

15.407
Fall 2002

Solution to Final Exam

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1. (a) **False**. Maximizing firm's market value is equivalent to investing in projects with positive and greatest NPV. NPV of projects with high expected returns can be negative.

   (b) **False**. The arguments of market efficiency and no arbitrage imply that the NPV of both types of bonds should be zero. There is no way to tell if one is a better investment than the others without considering other factors such as interest rate risk or re-investment opportunity.

   (c) **False**. Growth stocks are companies with access to positive present value of growth opportunities. How these companies pay their dividends depends on their payout ratio and dividend policy.

   (d) **False**. Recall the futures pricing formula

   $$F = S_0(1 + r - \hat{y})^T$$

   If the convenience yield is greater than the interest rate, futures price for long-term contracts can be lower than that for short-term contracts.

   (e) **Uncertain**. For European options, the put-call parity

   $$C = P + S - \mathrm{PV}(K) = P + S - \mathrm{PV}(S)$$

   can easily show that the call options are more valuable if the interest rate is positive. Such relationship may not hold for American options when the options can be exercised early.

   (f) **False**. Diversification can reduce risk whenever the asset returns are not perfectly correlated.

   (g) **False**. Assets with more volatile returns but low correlation with the market may earn a low expected return according to CAPM.

   (h) **False**. The correct discount rate should be the cost of capital corresponds to the riskiness of the projects.

Grade comments: I took points off for inaccurate comments or unclear answers. The answers of true or false would not affect the final credits. It is the explanation that counts.

2. (a) The present value of the liability is:

$$PV = 9.756 + 9.518 + 9.286 = 28.56 \text{ million}$$

(b) The duration of the liability is:

$$D = \frac{1}{28.56}(9.756 + 2 \times 9.518 + 3 \times 9.286) = 1.9835$$

(c) Let w_1, w_2 and $1 - w_1 - w_2$ be the amount invested in the 1-year, 2-year and 3-year STRIPS respectively. To eliminate interest rate risk, the value change of this portfolio should be the same as the value change of the liability in response to interest rate change, i.e.

$$\begin{aligned} \Delta P &= \Delta L \\ P \times D_P \times \Delta r &= L \times D_L \times \Delta r \\ 20 \times (w_1 + 2w_2 + 3(1 - w_1 - w_2)) &= 28.56 \times 1.9835 \end{aligned}$$

As you can see, there are multiple solutions to this equation. So, you can form different portfolios tailored to your preferences to achieve your hedging purposes. If you want to be fancy, you can consider portfolios with maximum convexity to benefit yourself from interest fluctuation. This solution will only demonstrate the case when $w_1 = 0$. The equation will become:

$$\begin{aligned} 20 \times (2w_2 + 3(1 - w_2)) &= 56.65 \\ w_2 &= 0.1675 \end{aligned}$$

So, we should invest 16.75% in 2-year discount bonds and 83.25% in 3-year discount bonds.

(d) If interest rate increases by 0.10%, the value of the liability will decrease by:

$$\Delta L = L \cdot MD_L \cdot \Delta r = 28.56 \cdot \frac{1.9835}{1 + \frac{0.02486}{2}} \cdot 0.001 = 0.0560$$

The value of 2-year bond holdings will decrease by

$$\Delta B_{2-\text{yr}} = B_{2-\text{yr}} \cdot MD_{B_{2-\text{yr}}} \cdot \Delta r = 3.35 \cdot \frac{2}{1.01243} \cdot 0.001 = 0.0066$$

The value of 3-year bond holdings will decrease by

$$\Delta B_{3-\text{yr}} = B_{3-\text{yr}} \cdot MD_{B_{3-\text{yr}}} \cdot \Delta r = 16.65 \cdot \frac{3}{1.01243} \cdot 0.001 = 0.0493$$

So, the net liability will decrease by

$$\Delta L - (\Delta B_{2\text{yr}} + \Delta B_{3\text{yr}}) = 0.0560 - (0.0066 + 0.0493) \approx 0$$

Grade comments: Many students had trouble hedging the interest rate risk in part c. Points were taken off from any answer that did not fulfill the hedging requirement. In part d, points were taken off from answers that were not consistent with previous works. For example, if a student did not fully hedge his portfolio risk in part c but then suggested a minimal value change to the net liability, he would be heavily penalized.

3. Assume market premium is 8%, which is a conservative estimate from the historical stock performance. Also assume the risk-free rate is 1.5%, which is obtained from the 6-month T-Bill rate.

   (a) Beta's cost of capital is:

   $$r_{\text{Beta}} = 1.5 + 0.8 \times 8 = 7.9\%$$

   Since its ROE is greater than its cost of capital, Beta should expand.

   (b) Beta Inc. is a growth stock since its ROE is greater than its cost of capital.

   Grade comments: Most students did well in this question, except that some of them had forgotten to explain their choice of estimates for market premium and risk-free rates. That would cost them some points.

4. (a) Using the futures price formula,

   $$\begin{aligned} F &= S(1 + r - \hat{y})^T \\ 4800 &= 5000(1 + 0.0025 - \hat{y})^3 \\ \hat{y} &= 1.6015\% \end{aligned}$$

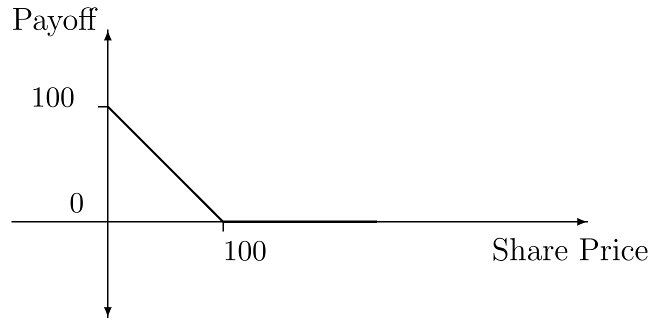
   So, the average monthly net convenience yield is 1.6015%.

   (b) You can purchase three-month futures on 10 tons of salmon.

   (c) Instead of purchasing futures, you can borrow \$50,000 from the bank at 0.25% per month for three months, use the proceeds to buy 10 tons of smoked salmon and earn the 1.8% net convenience yield. Three months later, you will have to return \$50,376 to the bank. Please note that this is also an arbitrage opportunity. So, as long as storage space allows, you can purchase more salmon and short the equivalent amount of futures to earn the arbitrage profit.

   Grade comments: Most students did well in this question too. However, in part c I took a few points off from answers that described an arbitrage strategy instead of hedging strategy.

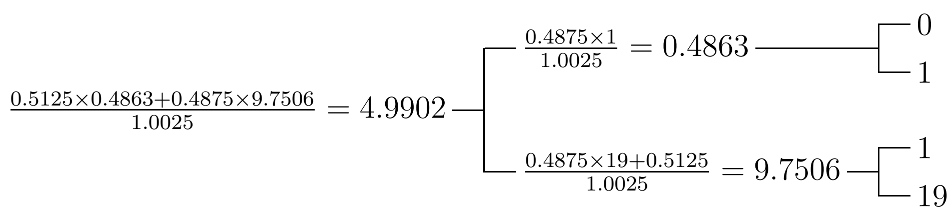
5. (a) The following graph shows the payoff of put at maturity:



(b) Using the formula of risk-neutral probabilities:

$$q = \frac{(1+r)-d}{u-d} = \frac{1.0025-0.9}{1.1-0.9} = 0.5125$$

The price of the put option can be described by the following tree



So, the value of the put is \$7.4762.

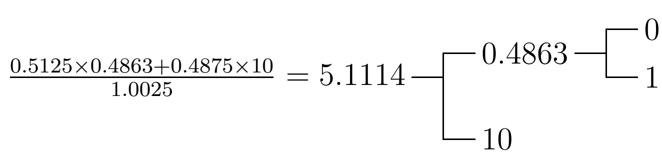
(c) Net gains/losses of the put option is defined as:

$$\text{Net Gain/Loss} = \text{Payoff} - \text{Put Price}(1.0025)^2$$

The following table shows the net gains/losses of the put option at maturity for different stock prices:

Stock price	121	99	99	81
Put option payoff	0	1	1	19
Net gain/loss	-5.0152	-4.0152	-4.0152	13.9848

(d) If the put is American-style, you can exercise early. When the stock price drops to \$90, exercising the put option early will give a payoff of \$10, which is larger than the value of the put option. So, it is optimal to exercise the option early when the stock price drops to \$90. The payoff of the put option can be described by the following tree:



So, the value of the put will increase to $5.1114.

Grade comments: Plenty of students were having trouble with part d. Partial credits were given to students who attempted to solve for the value of the American put option. For students who calculated the wrong option price in part b and therefore drew the wrong conclusion in part d, I took some points off from their answers as the difficulty of the question was changed substantially.

6. The optimal condition for a representative investors maximizing his utility over his consumption is given by:

$$r = \left(\frac{1}{\rho} - 1\right) + \frac{1}{\rho}\left(-\frac{c_0 u''(c_0)}{u'(c_0)}\right)\left(\frac{dc}{c_0}\right)$$

   When consumption growth $\frac{dc}{c_0}$ is expected to increase, real interest rate is expected to increase too.

   Grade comments: Points were deducted if the students failed to apply the interest rate model correctly.

7. James Bond is right only if the term structure follows Expectation Hypothesis. If the term structure follows Liquidity Preference Hypothesis, the market will demand a risk premium for holding longer-maturity bonds. When the risk premium is increasing with maturity and large enough, James will observe an upward-slopping yield curve even though the future spot rates are expected to drop.

   Grade comments: Points were deducted if the students were not able to explain the relationship between forward rates and future spot rates under different hypotheses.

8. (a) Expected return of new-economy fund is:

$$r_t = \beta(r_{Mt} - r_F) + r_F = 1(0.08) + 0.02 = 10\%$$

   (b) If CAPM holds, the market portfolio should be the tangent portfolio, which gives the highest return-to-risk raio. Let w be the weight on the market portfolio:

$$\begin{aligned} w(0.01) + (1-w)0.02 &= 0.08 \\ w &= 0.75 \end{aligned}$$

   So, the optimal portfolio should invest 75% in index fund and 25% in money market fund.

   (c) First of all, we have to figure out some of the important statistics about the new-economy fund. We know that:

$$\begin{aligned} R^2 &= \frac{\beta^2 \text{Var}(r_{Mt})}{\text{Var}(r_t)} \\ \text{Var}(r_t) &= \frac{0.2^2}{0.5} = 0.08 \end{aligned}$$

So, the standard deviation of the fund is 28.28%. α has a t-stat of 10.00, which suggests that α is quite statistical significant. The expected return of the fund is:

$$r_t = \alpha + \beta(r_{Mt} - r_F) + r_F = 0.02 + 0.08 + 0.02 = 12.00\%$$

We also know that

$$\begin{aligned} \beta &= \frac{\text{Cov}(r_{Mt}, r_t)}{\text{Var}(r_{Mt})} \\ \text{Cov}(r_{Mt}, r_t) &= 1 \times 0.2^2 = 0.04 \end{aligned}$$

So, the fund's covariance with the market return is 0.04. The addition of this new-economy fund will improve the portfolio frontier and change the composition of the tangent portfolio. The new optimal portfolio should be a combination of all three funds.

(d) Let w_1 and w_2 be the investment portfolio in index fund and new-economy fund respectively. So, the expected return of the portfolio is:

$$w_1 \times 0.1 + w_2 \times 0.12 + (1 - w_1 - w_2) \times 0.02 = 0.08$$

The variance of the portfolio is:

$$w_1^2 0.2^2 + w_2^2 0.08 + 2w_1 w_2 0.04$$

The goal is to minimize the variance of the portfolio such that its expected return is 8.0%. The Lagragian equation is set up as follows:

$$L = w_1^2 0.04 + w_2^2 0.08 + 0.08 w_1 w_2 + \lambda(0.08 w_1 + 0.1 w_2 - 0.06)$$

The first-order conditions are:

$$\begin{aligned} \frac{\partial L}{\partial w_1} &= 0.08w_1 + 0.08w_2 + 0.08\lambda = 0 \\ \frac{\partial L}{\partial w_2} &= 0.16w_2 + 0.08w_1 + 0.1\lambda = 0 \\ \frac{\partial L}{\partial \lambda} &= 0.08w_1 + 0.1w_2 - 0.06 = 0 \end{aligned}$$

Combining the three equations, we have:

$$\begin{aligned} w_1 &= 52.94\% \\ w_2 &= 17.65\% \\ 1 - w_1 - w_2 &= 29.41\% \end{aligned}$$

So, the optimal portfolio should invest 17.65% in the new-economy fund, 29.41% in the money market fund and 52.94% in index fund.

Grade comments: This question was a disaster as most students failed to realize that part b was supposed to be extremely easy whereas part d is the time-consuming one. Points were deducted if the students used the inefficient method (optimization) to approach part b. Partial credits were given in part d if the students were able to calculate the covariance between index and new-economy fund or the variance of the new-economy fund, or formulate the optimization problem correctly.

9. (a) Expected return of stock A:

$$\begin{aligned} r_A &= r_f + b_1\Pi_1 + b_2\Pi_2 \\ &= 0.02 + 1.2(0.065) + 0.0(-0.01) \\ &= 9.8\% \end{aligned}$$

Expected return of stock B:

$$\begin{aligned} r_B &= 0.02 + 0.8(0.065) - 0.5(-0.01) \\ &= 7.7\% \end{aligned}$$

(b) The expected return of the market portfolio is:

$$r_M = r_f + 0.065 = 8.5\%$$

The client can invest 50% in stock A and 50% in stock B. The expected return of the portfolio is:

$$r_p = 0.5 \times r_A + 0.5 \times r_B = 8.75\%$$

The portfolio's beta to the market is:

$$b_1^p = 0.5 \times 1.2 + 0.5 \times 0.8 = 1$$

This portfolio should be able to beat the market on average as its expected return is 0.25% higher than that of the market portfolio.

Grade comments: Students would not get full credits in part b unless they could show the composition of stocks in the client portfolio.

10. (a) The following table shows the after-tax cash flows of the project

Year	0	1	2	3	4	5
Sales Revenue		25	25	25	25	25
COGS		−6.30	−6.62	−6.95	−7.29	−7.66
Before-tax profit		18.70	18.39	18.05	17.71	17.34
After-tax profit		12.16	11.95	11.74	11.51	11.27
Depreciation tax shield		3.50	3.50	3.50	3.50	3.50
Investment	−50					
Free cash flow	−50	15.66	15.45	15.24	15.01	14.77

(b) The NPV of the project is:

$$\text{NPV} = \sum_{t=0}^{5} \frac{\text{CF}_t}{(1+r)^t} = \$7.87 \text{ million}$$

Since the NPV is greater than zero, we should take the project.

Grade comments: Some students had made an assumption that gold was purchased in the beginning of year 1. In that case, they had to treat gold as inventory (working capital) and proceed accordingly. Points were taken off if they failed to do so.