

MIT SLOAN SCHOOL OF MANAGEMENT

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Solution to Midterm Exam

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1. (a) **False.** The shareholders do not need to agree with the value of the firm's assets for the market value of the firm to be maximized. The shareholders can reconcile their differences in valuation by trading with the market.
 - (b) **False.** They should use the discount rate corresponding to the riskiness of the project.
 - (c) **False.** Price of a Treasury bond is determined by its future cash flow and spot rates. However, given enough number and variety of other Treasuries, you can use their yield-to-maturity to derive the spot rates.
 - (d) **False.** The market price of a share of stock equals the discounted value of the stream of future dividends per share.
2. Assume the savings occur in the beginning of year 1, 2 and 3 and the real estate will be bought in the end of year 3. Let A be the amount of the first deposit, the present value of savings is:

$$PV_{\text{saving}} = A + \frac{1.1}{1.04}A + \left(\frac{1.1}{1.04}\right)^2 A = 3.1764A$$

The present value of the real estate is:

$$PV_{\text{estate}} = 1000000$$

Equating the two equations and solving for A , we have

$$A = \$314821$$

So, the amount of the first deposit is \$314821.

3. (a) There are two ways to calculate the price of the February 2011 note. You can calculate the present value of the February 2011 note, which is:

$$\begin{aligned} B &= \frac{2.5}{\left(1 + \frac{3.65\%}{2}\right)^{\frac{4}{6}}} + \frac{2.5}{(1 + 0.01825)^{\frac{10}{6}}} + \cdots + \frac{102.5}{(1.01825)^{\frac{100}{6}}} \\ &= \frac{1}{(1.01825)^{\frac{4}{6}}} \left[2.5 + \frac{2.5}{1.01825} + \frac{2.5}{(1.01825)^2} + \cdots + \frac{102.5}{(1.01825)^{16}} \right] \\ &= 110.45 \end{aligned}$$

or find the dirty price from the given information:

- Clean price is 109:20 or 109.63
- Accrued interest is

$$\begin{aligned} & \text{coupon} \times \frac{\text{number of days from last coupon payment to settlement day}}{\text{number of days between coupon payments}} \\ &= 2.5 \times \frac{2}{6} \\ &= 0.8333 \end{aligned}$$

Since dirty price derived from the quote is:

$$\text{dirty price} = \text{clean price} + \text{accrued interest}$$

the dirty price is

$$109.63 + 0.83 = \$110.46$$

You would have to pay approximately \$110.46 for the February 2011 note. Coupon payment of \$2.5 will take place in February and August every year until February 2011, when, in addition to the coupon payment, you will receive the principal of \$100 as well.

- (b) The spot interest rate for February 2010 is:

$$\left(1 + \frac{3.65\%}{2}\right)^2 - 1 = 3.6833\%$$

The spot interest rate for February 2011 is:

$$\left(1 + \frac{3.77\%}{2}\right)^2 - 1 = 3.8055\%$$

- (c) Let the effective forward rate between February 2010 and February 2011 be f :

$$\begin{aligned} \left(1 + \frac{3.65\%}{2}\right)^{\frac{88}{6}} (1 + f) &= \left(1 + \frac{3.77\%}{2}\right)^{\frac{100}{6}} \\ f &= 4.7063\% \end{aligned}$$

So, the effective forward rate between February 2010 and February 2011 is 4.7063%.

- (d) The February 2010 Note has the shortest duration for the following reasons:

- Its maturity is shorter than the February 2011 Note.
- Its coupon rate is higher than the February 2011 Note.
- Regular coupon bond has a shorter duration than zero-coupon bond with the same maturity.

(e) The price of the February 2010 STRIPS as a function of its yield is:

$$B(y) = \frac{100}{\left(1 + \frac{y}{2}\right)^{\frac{88}{6}}} = 76.7011$$

The second derivative of $B(y)$ is:

$$\frac{d^2 B(y)}{dy} = \frac{1}{2} \frac{1}{2} \frac{88}{6} \frac{94}{6} \frac{100}{\left(1 + \frac{y}{2}\right)^{\frac{100}{6}}} = 4249.5315$$

So, the convexity of the February 2010 STRIPS is:

$$CX = \frac{1}{2B} \frac{d^2 B(y)}{dy} = \frac{4249.5315}{76.7011} = 27.70$$

Convexity is the curvature of the bond price as a function of the yield. It measures the second-order interest rate risk of bonds.

4. (a) Recall the equation

$$P = \frac{\text{EPS}_1}{r} + \text{PVGO}$$

So,

$$\begin{aligned} 22 &= \frac{1.6}{0.08} + \text{PVGO} \\ \text{PVGO} &= \$2 \end{aligned}$$

Since $\text{PVGO} > 0$, Plum Greek is a growth stock.

(b) Since you are not going to receive the first dividend payment, the Plum Creek Timber Company is worth to you:

$$\text{Market Price} - \text{PV}(\text{Dividend}) = 22 - \frac{1.4}{1.08} = \$20.70$$

But the bank only offers you \$19.50. So, this is a bad deal.

5. (a) Using the futures price formula

$$\begin{aligned} F &= S(1 + r - \hat{y})^T \\ 26.7 &= 27(1 + 0.03 - \hat{y})^{\frac{60}{365}} \\ \hat{y} &= 9.5712\% \end{aligned}$$

So, the net convenience yield is 9.5712%.

(b) As an oil refinery, you can

- Borrow money to buy oil and sell December futures on day 0.

- Earn the convenience yield of 12% over 60 days.
- After 60 days, Sell the oil, clear the futures position and repay the debt.

This is an arbitrage opportunity and you may want to maximize your position as much as possible. However, as you begin accumulating certain amount of oil, oil price and storage cost will rise, your net convenience yield will fall and your arbitrage profit will start to narrow.

(c) You need to purchase

$$0.8 \times \frac{1000000}{1000} = 800 \text{ futures contracts}$$