

Chapter 2

Present Value

Road Map

Part A Introduction to finance.

- Financial markets and financial decisions.
- Present value.

Part B Valuation of assets, given discount rates.

Part C Determination of discount rates.

Part D Introduction to corporate finance.

Main Issues

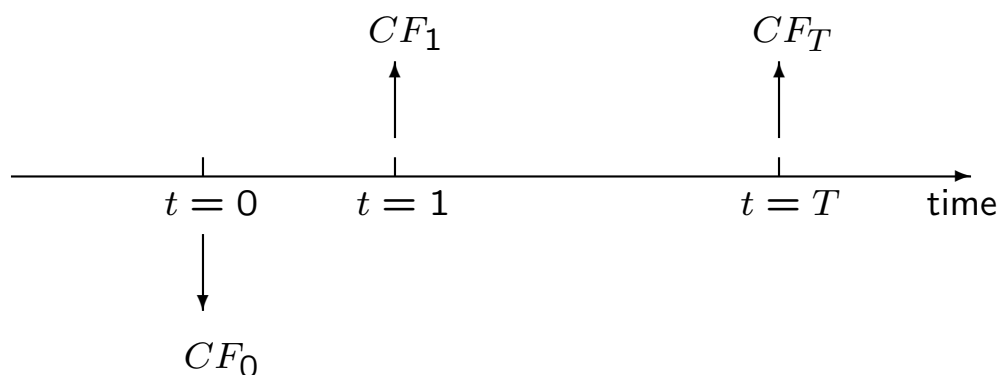
- Present Value
- Compound Interest Rates
- Nominal versus Real Cash Flows and Discount Rates
- Shortcuts to Special Cash Flows

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1 Valuing Cash Flows

“Visualizing” cash flows.



Example. Drug company develops a flu vaccine.

- Strategy A: To bring to market in 1 year, invest \$1 B (billion) now and returns \$500 M (million), \$400 M and \$300 M in years 1, 2 and 3 respectively.
- Strategy B: To bring to market in 2 years, invest \$200 M in years 0 and 1. Returns \$300 M in years 2 and 3.

Which strategy creates more value?

Problem. How to compare two CF *streams*.

Solution. For each CF stream, find a single number:

- Equal in value to the cashflow stream.
 - Valued on the same date (e.g. today) for all cash flows.
- Use this number to compare different CF streams.

1.1 Future Value (FV)

Current interest rate is r . How much will \$1 today be worth in one year?

- \$1 investable at a rate of return r , say, 4%.
- FV in 1 year is

$$FV = 1 + r = \$1.04.$$

- FV in t years is

$$\begin{aligned} FV &= \$1 \times (1+r) \times \cdots \times (1+r) \\ &= (1+r)^t. \end{aligned}$$

Example. Bank pays an annual interest of 4% on 2-year CDs and you deposit \$10,000. What is your balance two years later?

$$FV = 10,000 \times (1 + 0.04)^2 = \$10,816.$$

1.2 Present Value (PV)

We can ask the question in reverse.

Question. What is the PV of \$1 received a year from now?

- Consider putting away $1/(1+r)$ today. A year later receive:

$$\frac{1}{1+r} \times (1+r) = 1.$$

- The PV of \$1 received a year from now is:

$$\frac{1}{1+r}.$$

- The present value of FV dollars received t years from now is:

$$PV = \frac{FV}{(1+r)^t} = FV \times \frac{1}{(1+r)^t}.$$

Example. (A) \$10 M in 5 years or (B) \$15 M in 15 years. Which is better if $r = 5\%$?

$$PV_A = \frac{10}{1.05^5} = 7.84.$$

$$PV_B = \frac{15}{1.05^{15}} = 7.22.$$

Solution to Example. Flu Vaccine.

Assume that $r = 5\%$.

Strategy A:

Time	0	1	2	3
Cash Flow	-1,000	500.0	400.0	300.0
Present Value	-1,000	476.2	362.8	259.2
			Total PV	98.2

Strategy B:

Time	0	1	2	3
Cash Flow	-200	-200.0	300.0	300.0
Present Value	-200	-190.5	272.1	259.2
			Total PV	140.8

Firm should choose strategy B, and its value would increase by \$140.8 M.

2 Compound Interest Rates

2.1 Compound Interest vs. Simple Interest

- *Simple interest* - earn no interest on interest
- *Compound interest* - earn interest on interest.

Example: Compound and simple interest with $r = 10\%$.

Year	Simple			Compound		
	Starting Balance	Interest	Ending Balance	Starting Balance	Interest	Ending Balance
1	100	10	110	100.00	10.00	110.00
2	110	10	120	110.00	11.00	121.00
3	120	10	130	121.00	12.10	133.10
⋮	⋮	⋮	⋮	⋮	⋮	⋮
10	190	10	200	235.79	23.58	259.37
⋮	⋮	⋮	⋮	⋮	⋮	⋮
20	290	10	300	611.59	61.16	672.75
⋮	⋮	⋮	⋮	⋮	⋮	⋮
30	390	10	400	1586.31	158.63	1744.94

2.2 APRs and EARs

Typically compound interest is quoted using an *annual percentage rate* (APR) with an associated compounding interval.

Example. Fleet Bank's one-year CD offers 6% APR, with semi-annual compounding. If you invest \$10,000, how much money do you have at the end of one year? What is the actual annual rate of interest you earn?

- Quoted APR of $r_{\text{APR}} = 6\%$ is not the actual annual rate.
- It is only used to compute the 6-month interest rate as follows:

$$(6\%)(1/2) = 3\%.$$

- Investing \$10,000, at the end of one year you have:

$$10,000(1 + 0.03)(1 + 0.03) = 10,000(1.0609) = 10,609.$$

In the second 6-month period, you earn interest on interest.

- The actual annual rate, the *effective annual rate* (EAR), is

$$r_{\text{EAR}} = (1 + 0.03)^2 - 1 = 6.09\%.$$

Annual rates typically refer to EARs.

2.3 Compounding

Let r_{APR} be the annual percentage rate and k be the number of compounding intervals per year. One dollar invested today yields:

$$\left(1 + \frac{r_{\text{APR}}}{k}\right)^k$$

dollars in one year.

Effective annual rate, r_{EAR} is given by:

$$(1 + r_{\text{EAR}}) = \left(1 + \frac{r_{\text{APR}}}{k}\right)^k$$

or

$$r_{\text{EAR}} = \left(1 + \frac{r_{\text{APR}}}{k}\right)^k - 1.$$

Example. Suppose $r_{\text{APR}} = 5\%$:

k	Value of \$1 in a year	r_{EAR}
1	1.050000	5.0000%
2	1.050625	5.0625%
12	1.051162	5.1162%
365	1.051268	5.1267%
8,760	1.051271	5.1271%
\vdots	\vdots	\vdots
∞	$e^{0.05} = 1.051271$	5.1271%

Here, $e \approx 2.71828$.

3 Real vs. Nominal CFs and Rates

3.1 Real vs. Nominal Cash Flows

Question: If inflation is 4% per year and you have been promised \$1.04 in one year, what is this really worth next year?

What can \$1.04 buy next year given that prices will rise by 4%?

The *real or inflation adjusted* value of \$1.04 in a year is

$$\text{Real Cash Flow} = \frac{\text{Nominal Cash Flow}}{1 + \text{inflation}} = \frac{1.04}{1 + 0.04} = \$1.00.$$

In general, at annual inflation rate of i we have

$$(\text{Real Cash Flow})_t = \frac{(\text{Nominal Cash Flow})_t}{(1 + i)^t}.$$

3.2 Real vs. Nominal Interest (Discount) Rates

- Nominal interest rates - typical market rates
- Real interest rates - interest rates adjusted for inflation.

Example. \$1.00 invested at a 6% interest rate grows to \$1.06 dollars tomorrow. If inflation is 4% per year, then the real value is $\$1.06/1.04 = 1.019$. The real return is 1.9%.

$$1 + r_{\text{real}} = \frac{1 + r_{\text{nominal}}}{1 + i}.$$

Example. You are forecasting next year's revenues based on this year's sales of \$1 M. You expect 2% real growth in your sales (units). You expect inflation to be 4%. Suppose that the appropriate *nominal* discount rate is 5%. What is the present value of next year's sales revenue?

- Next year's *nominal* sales forecast:

$$1 \times 1.02 \times 1.04 = 1.0608.$$

$$PV = \frac{1.0608}{1.05} = 1.0103.$$

- Next year's *real* sales forecast:

$$1 \times 1.02 = 1.02.$$

Real discount rate:

$$r_{\text{real}} = \frac{1 + r_n}{1 + i} - 1 = \frac{1.05}{1.04} - 1 = 0.9615\%.$$

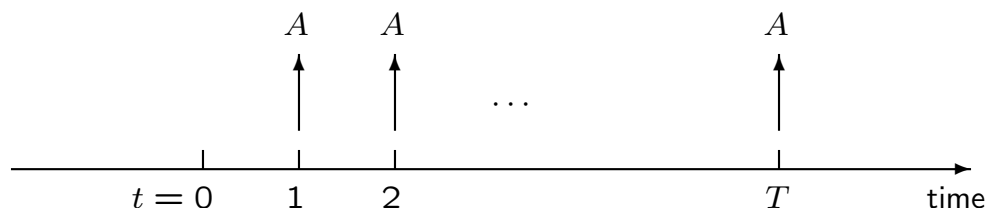
$$PV = \frac{1.02}{1.009615} = 1.0103.$$

Important Rules:

- Discount nominal CFs by nominal discount rates.
- Discount real CFs by real discount rates.

4 Shortcuts to Special Cash Flows

4.1 Annuity



Today is $t = 0$ and cash flow starts at $t = 1$.

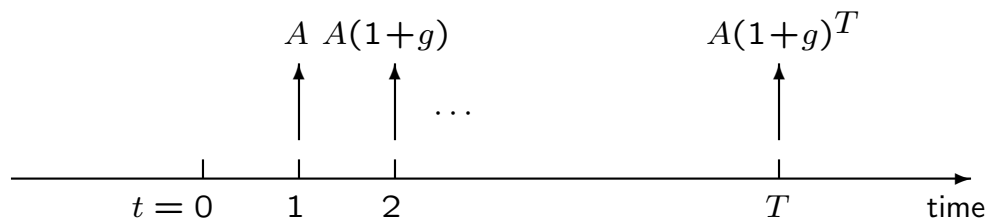
$$\begin{aligned} \text{PV (Annuity)} &= \frac{A}{1+r} + \frac{A}{(1+r)^2} + \dots + \frac{A}{(1+r)^T} \\ &= A \times \frac{1}{r} \left[1 - \frac{1}{(1+r)^T} \right]. \end{aligned}$$

Example. Insurance company selling an annuity of \$10,000 per year for 20 years. Suppose $r = 5\%$. What should the company sell it for?

$$\begin{aligned} PV &= 10,000 \times \frac{1}{0.05} \times \left(1 - \frac{1}{1.05^{20}} \right) \\ &= 10,000 \times 12.46 = 124,622.1. \end{aligned}$$

$$\text{FV (Annuity)} = \text{PV (Annuity)} \times (1+r)^T.$$

4.2 Annuity with Growth



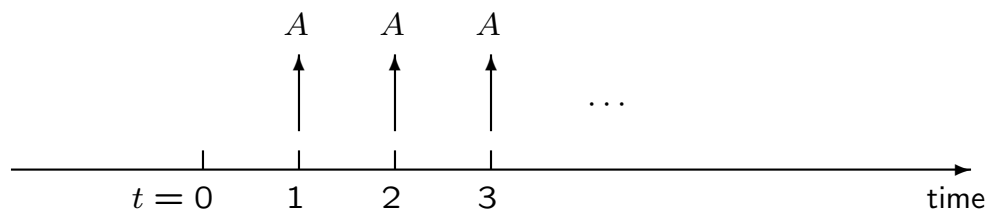
PV (growing annuity)

$$\begin{aligned}
 &= A \times \left[\frac{1}{1+r} + \frac{1+g}{(1+r)^2} + \dots + \frac{(1+g)^{T-1}}{(1+r)^T} \right] \\
 &= A \times \begin{cases} \frac{1}{r-g} \left[1 - \left(\frac{1+g}{1+r} \right)^T \right] & \text{if } r \neq g \\ \frac{T}{1+r} & \text{if } r = g. \end{cases}
 \end{aligned}$$

Example. Saving for retirement - Suppose that you are now 30 and would like \$2 million at age 65 for your retirement. You would like to save each year an amount that grows by 5% each year. How much should you start saving now, assuming that $r=8\%$?

$$A = \frac{2,000,000}{308.977} = 6,472.97.$$

4.3 Perpetuity



A perpetuity is an annuity with infinite maturity.

Example. You just won the lottery and it pays \$100,000 a year for 20 years. Are you a millionaire? Suppose that $r = 10\%$.

$$\begin{aligned} PV &= 100,000 \times \frac{1}{0.10} \left(1 - \frac{1}{1.10^{20}} \right) \\ &= 100,000 \times 8.514 = 851,356. \end{aligned}$$

- What if the payments last for 50 years?

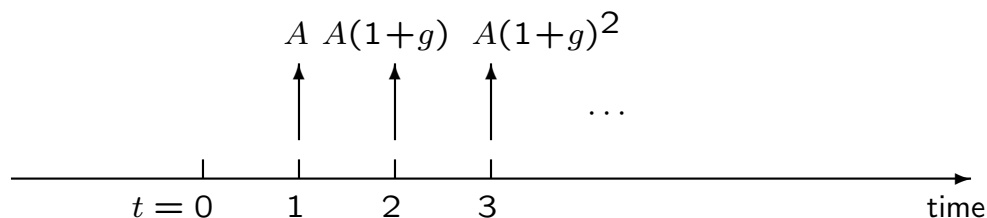
$$\begin{aligned} PV &= 100,000 \times \frac{1}{0.10} \left(1 - \frac{1}{1.10^{50}} \right) \\ &= 100,000 \times 9.915 = 991,481. \end{aligned}$$

- How about forever - a perpetuity?

$$PV = 100,000/0.10 = 1,000,000.$$

$$PV \text{ (Perpetuity)} = \frac{A}{r}.$$

4.4 Perpetuity with Growth



$$\text{PV (Perpetuity with growth)} = \frac{A}{r - g}$$

where A is the first CF one period from now.

Example. Super Growth Inc. will pay an annual dividend next year of \$3. After analyzing the company, you expect the dividend after that to grow at the rate of 5% per year forever. Also for companies of this risk class, the expected return is 10%. What should be Super Growth's price per share?

$$\begin{aligned} \text{PV} &= \frac{3}{1.10} + \frac{3(1 + 0.05)}{1.10^2} + \frac{3(1 + 0.05)^2}{1.10^3} + \dots \\ &= \frac{3}{0.10 - 0.05} \\ &= 60. \end{aligned}$$

Example. Mortgage calculation in the U.S.

- Pay 20% down payment, and borrow the rest from the bank using the property as collateral
- Pay a fixed monthly payment for the life of the mortgage
- Have the option to prepay the mortgage anytime before the maturity date of the mortgage.

Suppose that you bought a house for \$500,000 with \$100,000 down payment and financed the rest with a thirty-year fixed rate mortgage at 8.5% APR compounded monthly.

- The monthly payment M is determined by

$$\begin{aligned}
 400,000 &= \sum_{t=1}^{360} \frac{M}{[1 + (0.085/12)]^t} \\
 &= \frac{M}{(0.085/12)} \left\{ 1 - \frac{1}{[1 + (0.085/12)]^{360}} \right\} \\
 &= \frac{M \times (0.9212)}{(0.085/12)}.
 \end{aligned}$$

$$M = \$3,075.65.$$

- Your effective annual interest rate (EAR)

$$[1 + (0.085/12)]^{12} - 1 = 1.08839 - 1 = 8.839\%.$$

The monthly payments are as follows:

t (month)	Principal	Interest	Sum	Remaining P.
1	242.37	2833.33	3075.7	399,757.63
2	244.08	2831.62	3075.7	399,513.55
3	245.81	2829.89	3075.7	399,267.74
⋮	⋮	⋮	⋮	⋮
120	561.29	2514.42	3075.7	354,415.49
121	565.26	2510.44	3075.7	353,850.23
⋮	⋮	⋮	⋮	⋮
240	1309.27	1766.43	3075.7	248,068.95
241	1318.54	1757.16	3075.7	246,750.41
⋮	⋮	⋮	⋮	⋮
359	3032.60	43.10	3075.7	3,054.07
360	3054.07	21.63	3075.7	0.00

- Total monthly payment is the same for each month.
- The percentage of principal payment increases over time.
- The percentage of interest payment decreases over time.

5 Homework

Readings:

- BM Chapter 3.

Assignment:

- Problem Set 1.