Chapter 5

Forwards and Futures

Road Map

Part A Introduction to finance.

Part B Valuation of assets, given discount rates.

- Fixed income securities.
- Common stocks.
- Forwards and futures.
- Options.

Part C Determination of discount rates.

Part D Introduction to corporate finance.

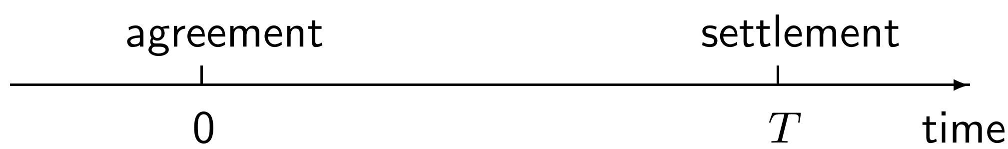
Main issues

- Forwards and Futures
- Forward and Futures Prices
- Hedging Financial Risk Using Forwards/Futures

Contents

1 Forward Contracts

Definition: A forward contract is a commitment to purchase at a future date a given amount of a commodity or an asset at a price agreed on today.



- The price fixed now for future exchange is the forward price.
- The party with a "long position" will be the buyer of the underlying asset or commodity.

Features of forward contracts:

- custom tailored
- traded over the counter (not on exchanges)
- no money changes hands until maturity
- non-trivial counter-party risk.

Example. Consider a 3-month forward contract for 1,000 tons of soybean at a forward price of \$165/ton. The long side is committed to buy 1,000 tons of soybean from the short side in three months at the price of \$165/ton.

2 Futures Contracts

Forward contracts have two limitations:

(a) illiquidity

(b) counter-party risk.

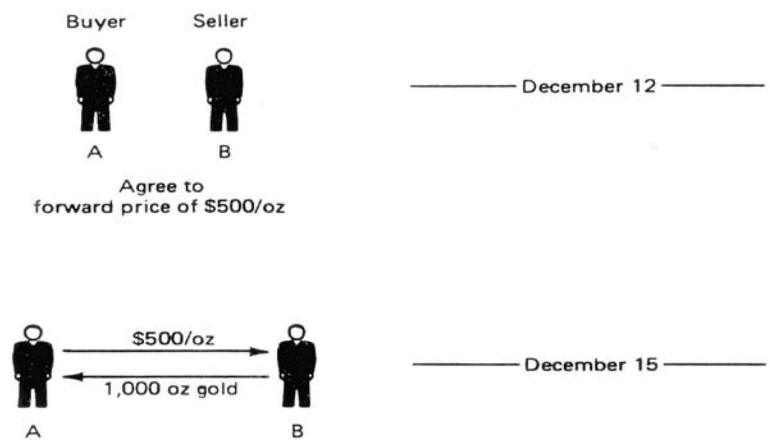
Futures contracts are designed to address these two limitations.

Definition: A futures contract is an exchange-traded, standardized, forward-like contract that is marked to the market daily. This contract can be used to establish a long (or short) position in the underlying asset.

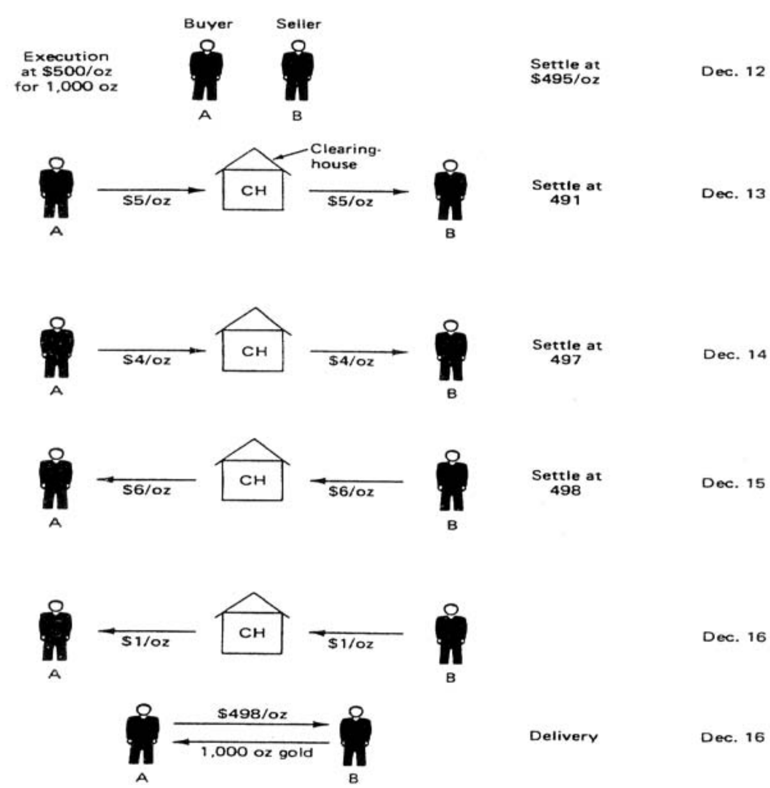
Features of futures contracts:

- Standardized contracts:
  (1) underlying commodity or asset
  (2) quantity
  (3) maturity.
- Exchange traded
- Guaranteed by the clearing house — no counter-party risk
- Gains/losses settled daily
- Margin account required as collateral to cover losses.

A Forward Contract



A Futures Contract



Example. Yesterday, you bought ten December live-cattle contracts at CME, at a price of \$0.7455/lb.

- Contract size 40,000 lb.
- Agreed to buy 400,000 pounds of live cattle in December.
- Value of position yesterday:

  $$(0.7455)(10)(40,000) = \$298,200.$$
- No money changed hands.
- Initial margin required (5%-20% of contract value).

Today, the futures price closes at \$0.7435/lb, 0.20 cents lower. The value of your position is

$$(0.7435)(10)(40,000) = \$297,400$$

a loss of \$800.

- Standardization makes futures liquid.
- Margin and marking to market reduce default risk.
- Clearing-house guarantee reduces counter-party risk.

Forward and futures contracts are derivative securities because

- payoffs determined by prices of the underlying asset
- zero net supply.

3 Forward and Futures Prices

Question: What determines forward and futures prices?

Answer: Forward/futures prices are linked to spot prices.

Contract	Spot at t	Forward	Futures
Price	S_t	F	H

Ingnoring differences between forward and futures, we have

$$F \simeq H.$$

Two ways to buy the underlying for date T:

1. Buy forward or futures contract of maturity T.
2. Buy the underlying now and store it until T.

Difference between buy-and-store from forward/futures:

a. Cost of storing (for commodities)

b. Benefits from storing
   - Convenience yield (for commodities)
   - Dividends (for financials).

By arbitrage, the costs of these two approaches must equal:

$$F \simeq H = (1 + r_F)S_0 + \text{FV (net cost of storing)}.$$

3.1 Commodities

1. Gold.

   - Easy to store—negligible cost of storage.
   - No dividends or benefits.

   Two ways to buy gold for T:

   - Buy now for S_0 and hold until T.
   - Buy forward, pay F and take delivery at T.

   No-arbitrage requires that

   $$F = S_0(1 + r_F)^T \simeq H.$$

   Example. Gold quotes on 2001.08.02 are

   - Spot price (London fixing) \$267.00/oz
   - October futures (CMX) \$269.00/oz.

   The implied 2-month interest rate is $r_F = 4.58\%$.

2. Gasoline.

   - Costly to store.
   - Additional benefits, *convenience yield*, for holding physical commodity (over holding futures).
   - Not held for long-term investment (unlike gold), but mostly held for future use.

Let the percentage holding cost be c and convenience yield be y.

We have

$$\begin{aligned} F &= S_0\left[1 + r_F - (y - c)\right]^T \\ &= S_0(1 + r_F - \widehat{y})^T \\ &\simeq H \end{aligned}$$

where

$$\widehat{y} = y - c$$

is the net convenience yield.

Example. Gasoline quotes on 2001.08.02: spot price is 0.7760, Feb 02 futures price is 0.7330, and 6-month interest rate is 3.40%.

Annualized net convenience yield is: $\widehat{y} = 14.18\%$.

For commodity futures:

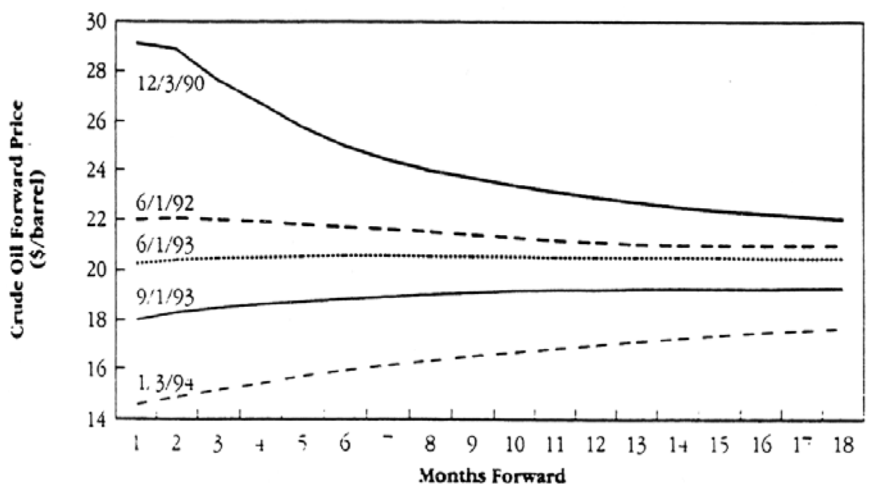
1. *Contango* means:
   (a) spot prices are lower than futures prices, and/or
   (b) prices for near maturities are lower than for distant.

2. *Backwardation* means:
   (a) spot prices are higher than futures prices, and/or
   (b) prices for near maturities are higher than for distant.

Backwardation occurs if convenience yield exceeds storage cost:

$$\widehat{y} - r_F = y - c - r_F > 0.$$

Crude oil forward price curves for selected dates



3.2 Financials

For financial futures, the underlying are financial assets. Financials have the following features:

- No cost to store (the underlying asset).
- Dividend or interest on the underlying.

1. Stock index futures.
    - Underlying are bundles of stocks — S&P, Nikkei, etc.
    - Futures settled in cash (no delivery).

    Let the dividend yield be d, then there is the following relation between the forward/futures price and spot price:

    $$F = S_0(1 + r_F - d)^T \simeq H.$$

    Deviations from this relation triggers index arbitrage.

    Example. S&P 500 closed at 1,220.75 on 2001.08.02 and S&P futures maturing in December closed at 1,233.50. Suppose the T-bill rate is 3.50%.

    The annual dividend yield is: $d = 0.33\%$ (lower than historic average).

Note:

- Since the underlying asset is a portfolio in the case of index futures, trading in the futures market is easier than trading in cash market.
- Thus, futures prices may react quicker to macro-economic news than the index itself.
- Index futures are very useful to market makers, investment bankers, stock portfolio managers:
  - hedging market risk in block purchases & underwriting
  - creating synthetic index fund
  - portfolio insurance.

Example. You have $1 million to invest in the stock market and you have decided to invest in a diversified portfolio. S&P seems a good candidate. How would you do this?

(a) One approach is to buy S&P in the cash market:

- buy the 500 stocks
- weights proportional to their market capitalization.

(b) Another way is to buy S&P futures:

- Put the money in your margin account
- Assuming S&P is at 1000 now, number of contract to buy:

  $$\frac{1000000}{(250)(1000)} = 4.$$

  (Value of a futures contract is $250 times the S&P index.)

As the S&P index fluctuates, the future value of your portfolio (in $M) would look as follows (ignoring interest payments and dividends):

S&P	Portfolio (a)	Portfolio (b)
450	0.90	0.90
500	1.00	1.00
550	1.10	1.10

2. Interest rate futures.

- Underlying assets are riskless or high grade bonds.

Forward prices are simply determined by forward interest rates.

Example. Consider a T-bond with annul coupon rate of 7% (with semi-annual coupon payments) that is selling at par. Suppose that the current short rate is 5%. What should be the 6-month forward price of the T-bond?

Consider the following strategy and its cash flow

Time	0	1	2	3	⋯
Buy T-bond	-100.0	3.5	3.5	3.5	⋯
Sell T-bill	100.0	-102.5	0	0	⋯
Net	0	- 99.0	3.5	3.5	⋯

This strategy allows one to lock in a purchase of the 7% T-bond 6-month later for $99. No arbitrage requires the current forward price to be $99.

In general, a bond's forward price is

$$F = S(1 + r - y)$$

where

- S is its spot price
- r is the spot interest rate
- $y = C/S$ is its coupon yield.

4 Hedging with Forwards and Futures

4.1 Hedging with Forwards

Hedging with forward contracts is simple, because one can tailor the contract to match maturity and size of position to be hedged.

Example. Suppose that you, the manager of an oil exploration firm, have just struck oil. You expect that in 5 months time you will have 1 million barrels of oil. You are unsure of the future price of oil and would like to hedge your position.

Using forward contracts, you could hedge your position by selling forward 1 million barrels of oil. Let S_t be the spot oil price at t (in months). Then,

Position	Value in 5 months (per barrel)
Long position in oil	S_5
Short forward position	$F - S_5$
Net payoff	F

Thus, in this case you know today exactly what you will receive 5 months from now. That is, the hedge is perfect.

4.2 Hedging with Futures

One problem with using forwards to hedge is that they are illiquid.

Thus, if after 1 month you discover that there is no oil, then you no longer need the forward contracts. In fact, holding just the forward contracts you are now exposed to the risk of oil-price changes.

In this case, you would want to unwind your position by buying these contracts. Given the illiquidity of forward contracts, this may be difficult and expensive.

To avoid problems with illiquid forward markets, one may prefer to use futures contracts.

Example. In the above example, you can sell 1 million barrels worth of futures. Suppose that the size of each futures contract is 1,000 barrels. The number of contract you want to short is

$$\frac{1,000,000}{1,000} = 1,000.$$

Example. Using interest rate futures to hedge a bond portfolio. We have \$10 million invested in government bonds and are concerned with highly volatile interest rate over the next six months. We decide to use the 6-month T-bond futures to protect the value of the portfolio. We have

- Duration of the bond portfolio is 6.80 years.
- Current futures price is $93\frac{2}{32}$ (for face value of \$100).
  - The T-bond to be delivered has a yield of 8.80% and duration is 9.20 years.
  - Each contract delivers \$100,000 face value of bonds.
  - Futures price for the total contract is \$93,062.50.

We should short the futures:

- If interest rate goes up, bond prices go down but a gain is made on the short position of futures.
- If interest rate goes down, bond prices go up but a loss is made on the short position of futures.

How many contracts to short? Match duration:

$$(\text{\# of contracts})(93,062.50)(9.20) = (10,000,000)(6.80).$$

Thus:

$$(\text{\# of contracts}) = \left(\frac{10,000,000}{93,062.50}\right)\left(\frac{6.80}{9.20}\right) = 79.42.$$

Since futures contracts are standardized, they may not perfectly match your hedging need. The following mismatches may arise when hedging with futures:

- Maturity mismatch
- Contract size mismatch
- Asset mismatch.

Thus, a perfect hedge is available only when

1. The maturity of futures matches that of the cash flow.
2. The contract has the same size as the position to be hedged.
3. The cash flow being hedged is linearly related to the futures.

In the event of a mismatch between the position to be hedged and the futures contract, the hedge may not be perfect.

Example. Continuing with the example of hedging oil 5 months from now. Suppose that you can only buy futures contracts that mature either 3 months from now or 6 months from now. Then, your hedge may not be perfect. Let

- S_t denote spot price at t
- $H_{t,T}$ denote the futures price at t with maturity T (in months).

Suppose that we use the 6-month futures to hedge. Ignoring marking to market, we have for each barrel:

Position	Value 5 months from today
Long position in oil	S_5
Long futures position	$H_{5,6} - H_{0,6}$
Hedged position	$S_5 - \left(H_{5,6} - H_{0,6}\right)$

Because the futures contract matures at $t = 6$, after 6 months $S_6 = H_{6,6}$.

But after only 5 months, the futures price does not equal the spot price then, S_5. Thus, the amount that you get then will not be exactly H_0.

4.3 Basis and Basis Risk

Definition: Basis refers to the difference between the futures price and the spot price.

Example. On 2001.08.02, S&P index closed at 1,220.75 while the December futures closed at 1,233.50. The basis (for December contract) is

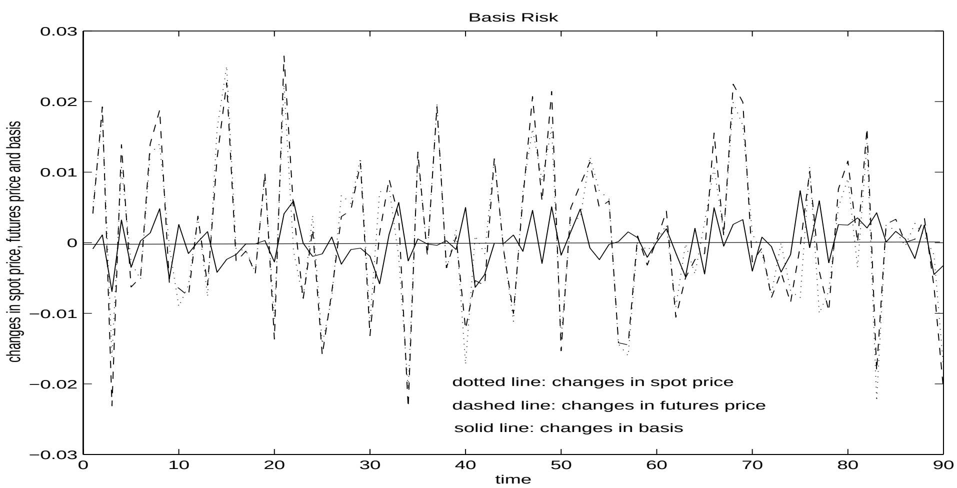
$$1233.50 - 1220.75 = 12.75.$$

As time passes, the basis changes.

When the underlying assets of the futures market and the cash market are identical, the basis converges to zero on the maturity date. Hence, using futures with matching underlying and maturity gives a perfect hedge.

Often one has to use instruments with basis not converging to zero on target date.

Definition: Basis risk refers to the uncertainty in the basis of a hedging instrument on the target date.



Basis risk arises from several reasons:

- mismatch of underlying asset

  Example. An investment bank shorts index futures to hedge the risk in underwriting a large stock issue.

- mismatch of maturity

  Example. Roll over short contracts to hedge long term risks.

Basis risk leads to imperfect hedge.

4.4 Minimum Variance Hedge

Definition: The hedge ratio is the number of futures used to hedge a unit exposure to the risk of spot price.

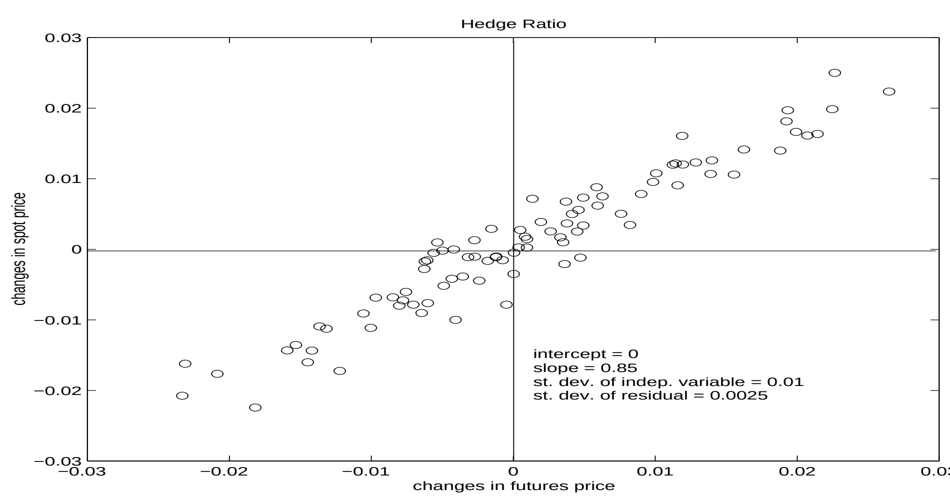
With basis risk, how do we choose optimal hedge ratio?

The spot price and the futures price have the regression relation:

$$\Delta S_t = a + b\Delta H_t + e_t$$

where

$$\begin{aligned} 0 &= \mathrm{E}[e_t] \\ 0 &= \mathrm{Cov}[\Delta H_t, e_t] \\ b &= \frac{\mathrm{Cov}[\Delta S_t, \Delta H_t]}{\mathrm{Var}[\Delta H_t]} \end{aligned}$$



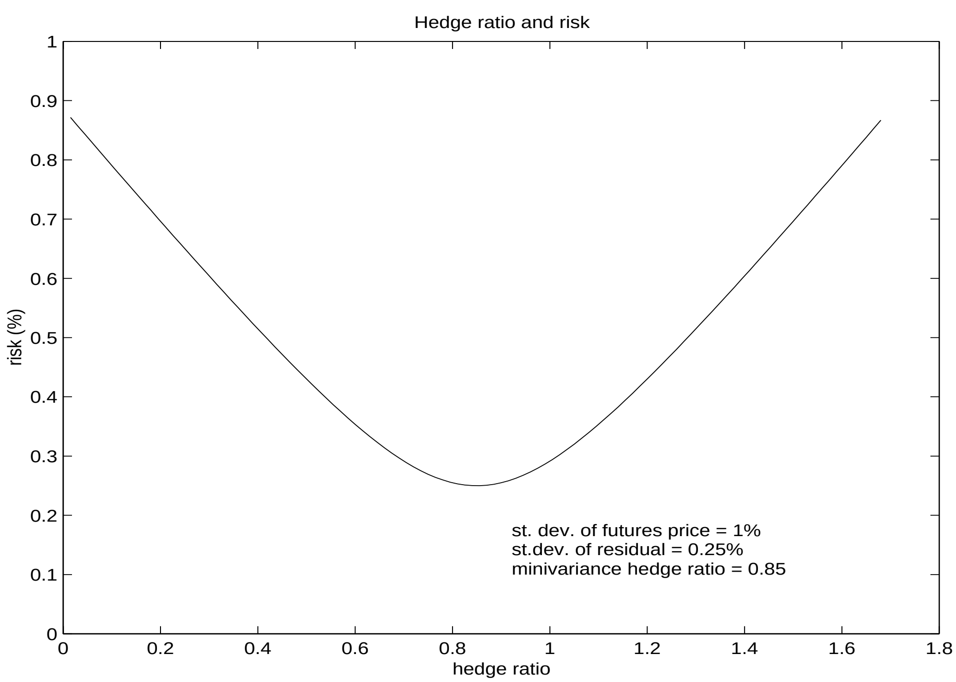
Defintion: The minimum variance hedge ratio is the hedge ratio that gives the minimum variance for the value of the hedged position.

For an arbitrary hedge ratio h, the variance of hedged position is

$$\text{Var}[\Delta S_t - h\Delta H_t] = \text{Var}[(b-h)\Delta H_t + e_t]$$
$$= (b-h)^2\text{Var}[\Delta H_t] + \text{Var}[e_t].$$

The variance is minimized with the hedge ratio:

$$h^* = b = \frac{\text{Cov}[\Delta S_t, \Delta H_t]}{\text{Var}[\Delta H_t]}.$$



5 Homework

Readings:

- BKM Chapters 22, 23.
- BM Chapter 27.

Assignment:

- Problem Set 4.