Chapter 8

Time Value of Money

Road Map

Part A Introduction to Finance.

Part B Valuation of assets, given discount rates.

Part C Determination of discount rates.

- Historical asset returns.
- Time value of money.
- Risk.
- Portfolio theory.
- Capital Asset Pricing Model (CAPM).
- Arbitrage Pricing Theory (APT).

Part D Introduction to corporate.

Main Issues

- Theroy of Real Interest Rates
- Nominal Interest Rates
- Term Structure Hypotheses

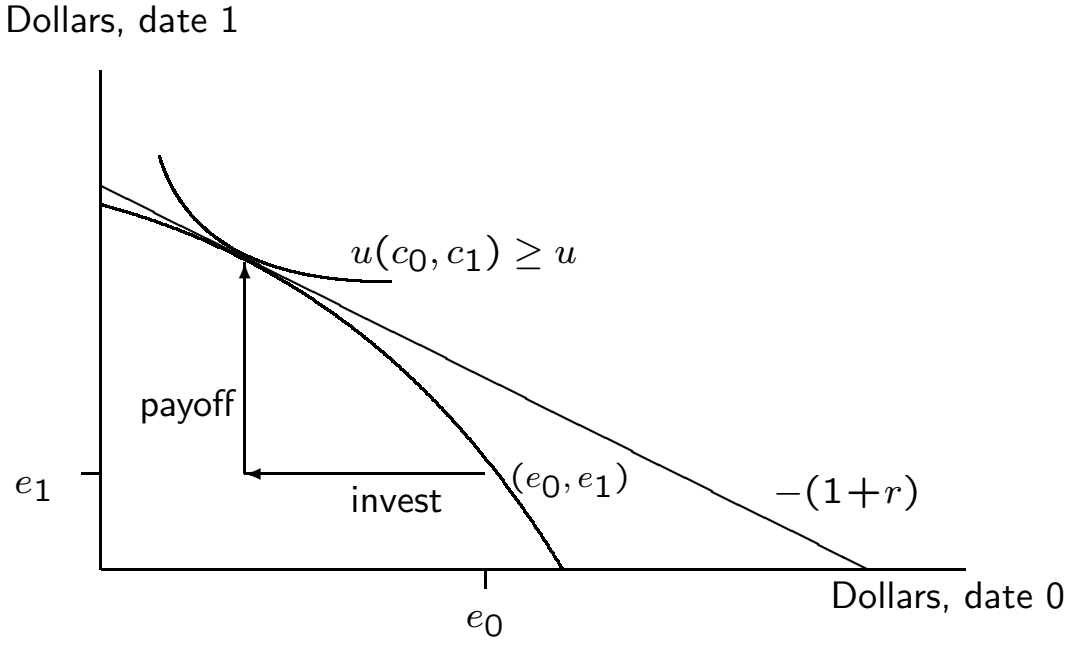
Contents

1 Theory of Real Interest Rates

Real interest rates are determined by supply and demand of funds in the economy.

Three factors in determining real interest rates:

1. Aggregate endowments
2. Aggregate investment opportunities
3. Aggregate preferences for different consumption path.



Consider a representative investor:

- has endowment of (e_0, e_1)
- faces a bond market with interest rate r.

He maximizes his utility over his consumption now and later:

$$\begin{aligned} \max \quad & u(c_0) + \rho u(c_1) \\ \text{s.t.} \quad & c_0 = e_0 - b \\ & c_1 = e_1 + (1+r)b \end{aligned}$$

where b is his bond holding, $u' > 0$ and $u'' < 0$.

The optimality condition is

$$u'(c_0) = (1+r)\rho u'(c_1)$$

or (for $c_1 = c_0 + dc$)

$$r = \frac{u'(c_0)}{\rho u'(c_1)} - 1 \approx \left(\frac{1}{\rho} - 1\right) - \frac{1}{\rho}\frac{c_0 u''(c_0)}{u'(c_0)}\frac{dc}{c_0}.$$

Thus, the real interest rate is given by

$$r = \left(\frac{1}{\rho} - 1\right) + \frac{1}{\rho}\left[-\frac{c_0 u''(c_0)}{u'(c_0)}\right]\left(\frac{dc}{c_0}\right).$$

The interest rate is determined by

1. Investors' time-impatience coefficients ρ
2. Rate of consumption growth
3. Relative sensitivity of marginal utility to consumption.

The interest rate is higher if

- investors are less patient for future consumption (smaller ρ)
- consumption grows faster
- marginal utility diminishes faster as consumption increases.

The real interest rate can be negative if consumption is expected to decrease significantly.

More generally, consumption growth can be uncertain. Investors maximize their expected utility over many periods:

$$\begin{aligned} \max \ & \mathrm{E}\left[\ \sum_{t=0}^{T} \rho^t \, u(c_t) \ \right] \\ \text{s.t.} \quad & c_0 = e_0 - b_1 - b_2 - \cdots b_T \\ & c_t = e_t + (1 + r_t) b_t, \quad t = 1, \ldots, T \end{aligned}$$

where $(b_1, \ldots, b_T)$ is his holdings of discount bonds, $(e_1, \ldots, e_T)$ future endowments, $(c_1, \ldots, c_T)$ his future consumption, both can be uncertain.

We then have

$$r_t = \rho^{-t} \, \mathrm{E}\left[\frac{u'(c_0)}{u'(c_t)} \right] - 1$$

where $t = 1, 2, \ldots$

$\{r_1, r_2, \ldots, r_T, \ldots\}$ gives the term structure of interest rates.

Thus, the term structure of interest rates is determined by

- Investors' impatience for future consumption
- Expected consumption growth at different horizons
- Their sensitivity of marginal utility to consumption growth.

2 Nominal Interest Rates

Coupon payments and principal payments on bonds are often in nominal terms. The interest rates bonds offer are then in nominal terms.

The link between is given by the following relation:

$$1 + r_{\text{nominal}} = (1 + r_{\text{real}})\,(1 + i)$$

where i is the rate of inflation.

Thus, two factors determine nominal interest rates:

1. Real interest rates
2. Expectations of future inflation.

The two factors are not necessarily independent.

Real interest rates can be negative. How about nominal rates?

3 Term Structure Hypotheses

3.1 Expectations Hypothesis

Expectations Hypothesis: Forward rates are unbiased predictors of expected future spot rates.

$$f_t = \mathrm{E}\left[r_1(t)\right].$$

Implications of Expectations Hypothesis:

- Slope of the term structure reflects the market's expectations of future short-term interest rates.
  - Upward slopping term structure reflects market expectations of increasing short-term interest rates over time.
  - Downward slopping term structure reflects market expectations of decreasing short-term interest rates over time.

- Expected rate of return on rolling over short bonds equals return on long bonds.

Example. Suppose that the current spot rates are:

Maturity	1	2	3	4	$\cdots$
Spot interest rate	0.030	0.040	0.045	0.050	
Forward interest rates	0.030	0.050	0.055	0.060	

The term structure of interest rates is upward slopping.

Under Expectations Hypothesis, an upward-slopping yield curve implies market's expectation of future increases in interest rates.

(1) What is the (locked-in) return on a 2-year bond?

$$(1+r_2(0))^2 - 1 = (1.04)^2 - 1 = 8.16\%.$$

(2) What is the expected return of rolling over 1-year bonds?

$$\begin{aligned} \mathrm{E}\left[(1+r_1(0))(1+r_1(1))\right] - 1 &= (1+r_1(0))(1+\mathrm{E}[r_1(1)]) - 1 \\ &= (1+r_1(0))(1+f_2(0)) - 1 \\ &= (1.03)(1.0501) - 1 = 8.16\%. \end{aligned}$$

Rolling over short bonds gives the same expected rate of return as holding long bonds.

3.2 Liquidity Preference Hypothesis

Liquidity Preference Hypothesis: Investors regard long bonds as riskier than short bonds.

The forward rates (or long bond yields) contain a risk premium:

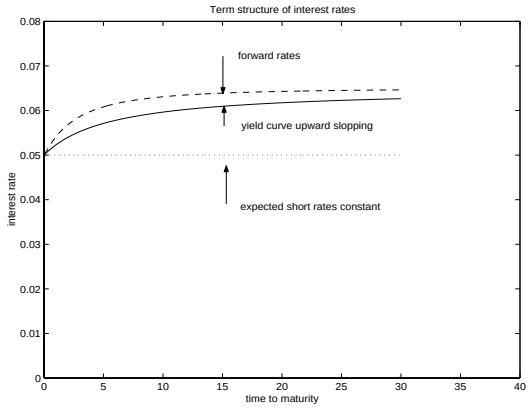
$$f_t = \mathrm{E}[r_1(t)] + \pi_t$$

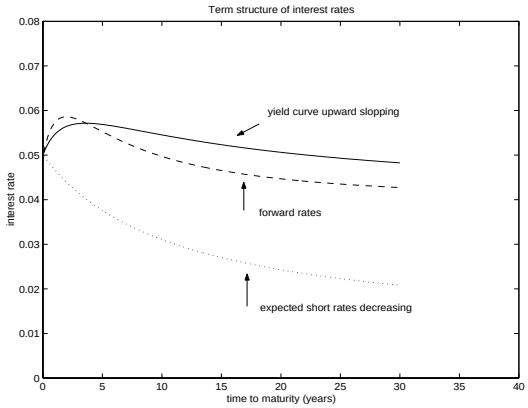
where π_t is the risk premium for discount bonds maturing at t.

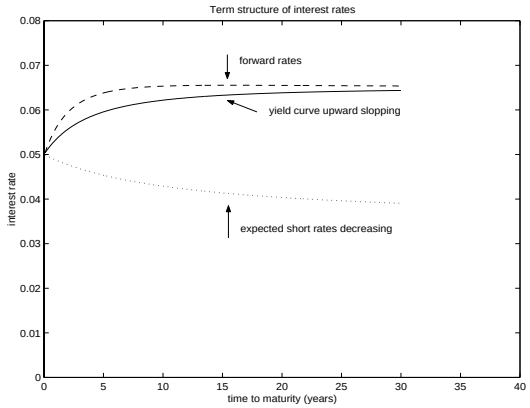
Implications of Liquidity Preference Hypothesis:

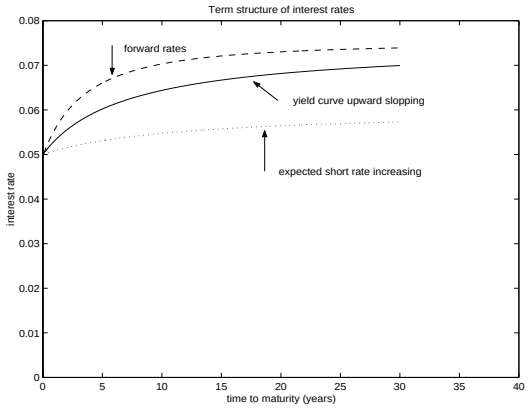
- Investors in long bonds expect, and on average receive, higher returns than investors in short bonds.
- Forward rate on average "overpredict" future short-term rates.
- Term structure reflects (a) expectations of future interest rates and (b) risk premium demanded by investors on long bonds.
- The term structure will slope upward even when future short-term rates are expected to be constant.

Yield curves with different expectations of future short rates

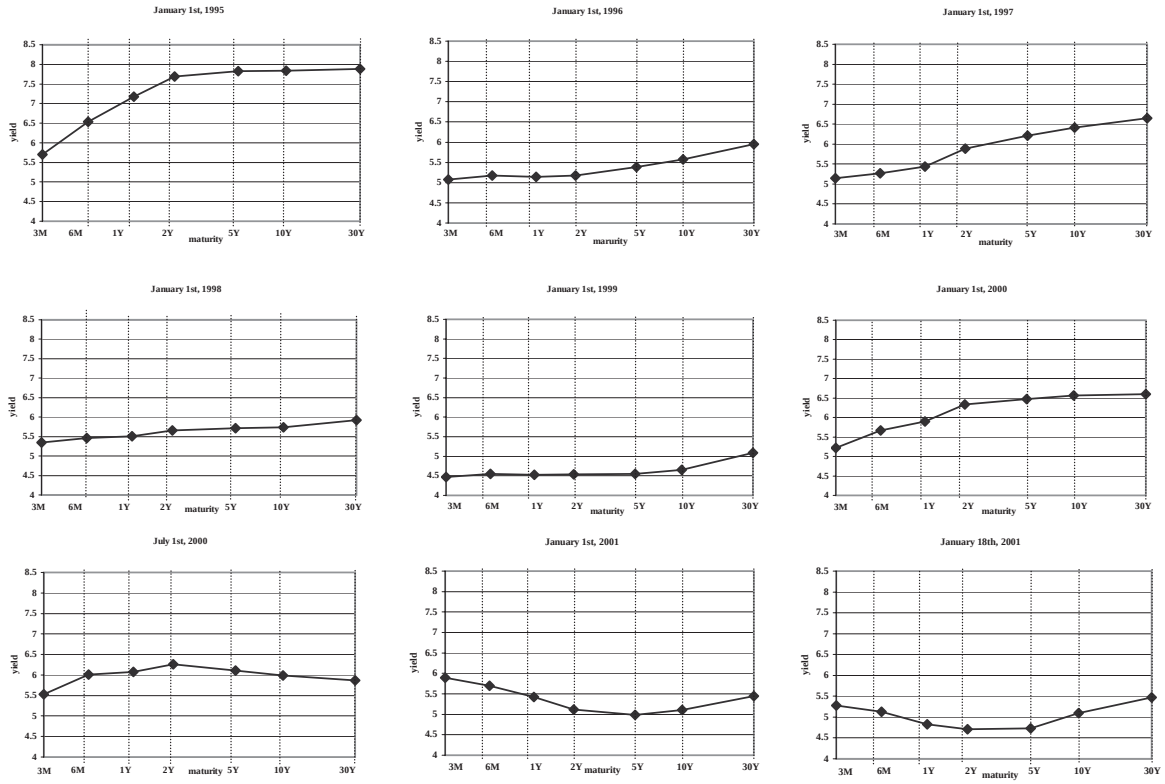








Treasury yield curves



Average rates of return on treasuries, 1926 – 1996

(Source: Ibbotson Associates, 1997 Yearbook)

	Long-term	Bills
Nominal	5.4%	3.8%
Real	2.4%	0.7%

(Inflation is 3.2%.)

4 Homework

Readings:

- BKM Chapter 5.1, 15.3, 15.4.
- BM Chapter 24.1, 24.4.

Assignment:

- Problem Set 6.