

# Chapter 14

## Capital Budgeting

### Road Map

**Part A** Introduction to finance.

**Part B** Valuation of financial assets, given discount rates.

**Part C** Determination of discount rates.

**Part D** Introduction to corporate finance.

- Efficient Market Hypothesis (EMH).
- Capital budgeting.
- Real options.
- Financing.

### Main Issues

- NPV Rule
- Cash Flow Computations
- Discount Rates — Using CAPM and APT
- Alternatives to NPV Rule

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# 1 NPV Rule

The objective of investment decisions is to increase the firm's current market value. Let the cash flow of an investment (a project) be

$$\{CF_0, CF_1, \dots, CF_t\}.$$

Its current market value is

$$NPV = CF_0 + \frac{CF_1}{1 + r_1} + \dots + \frac{CF_t}{(1 + r_t)^t}.$$

This is the addition to the firm's market value by the project (recall value additivity).

## Investment Criteria:

1. For a single project, take it if and only if its NPV is positive.
2. For many *independent* projects, take all those with positive NPV.
3. For *mutually exclusive* projects, take the one with positive and highest NPV.

In order to compute the NPV of a project, we need to analyze

1. Cash flows
2. Discount rates
3. Strategic options.

These three factors are the focus of the rest of this chapter.

## 2 Cash Flow Calculations

### Main Points:

1. Use cash flows, not accounting earnings.
2. Use after-tax cashflows.
3. Use cash flows attributable to the project (compare firm value with and without the project):
  - Use incremental cash flows
  - Forget sunk costs: bygones are bygones
  - Include investment in working capital as capital expenditure
  - Include opportunity costs of using existing equipment, facilities, etc.
  - Correct for biases from fighting for resources inside firm.

In what follows, all cash flows are attributable to the project.

$$\begin{aligned} CF &= [\text{Project Cash Inflows}] - [\text{Project Cash Outflows}] \\ &= [\text{Operating Revenues}] \\ &\quad - [\text{Operating Expenses without depreciation}] \\ &\quad - [\text{Capital Expenditures}] \\ &\quad - [\text{Income Taxes}]. \end{aligned}$$

Defining operating profit by

$$\begin{aligned} \text{Operating Profit} &= \text{Operating Revenues} \\ &\quad - \text{Operating Expenses w/o Depreciation} \end{aligned}$$

The income taxes are

$$\begin{aligned} [\text{Project Income Taxes}] &= [\text{Tax Rate}][\text{Operating Profit}] \\ &\quad - [\text{Tax Rate}][\text{Depreciation}] \end{aligned}$$

Note that accounting depreciation does affect cash flows because it reduces the company's tax bill.

Let  $\tau$  denote the "effective" tax rate. Then

$$\begin{aligned} CF &= (1 - \tau)[\text{Operating Profits}] - [\text{Capital Expenditures}] \\ &\quad + (\tau)[\text{Depreciation}]. \end{aligned}$$

## 2.1 Use Cash Flows, Not Accounting Earnings

**Example.** Capital Expenditure and Accounting Earnings vs. Cash Flows.

A machine purchased for \$1,000,000 with a life of 10 years generates annual revenues of \$300,000 and operating expenses of \$100,000. Assume that machine will be depreciated over 10 years using straight-line depreciation. The corporate tax rate is 40%.

Date	Accounting Earnings Before Tax	Accounting Earnings After Tax	Cash Flow After-tax
0	0	0	- 1,000,000
1	$300,000 - 100,000 - 100,000 = 100,000$	$(1-0.4)(100,000) = 60,000$	$(1-0.4) (300,000-100,000) + 40,000 = 160,000$
2	100,000	60,000	160,000
3	100,000	60,000	160,000
4	100,000	60,000	160,000
5	100,000	60,000	160,000
6	100,000	60,000	160,000
7	100,000	60,000	160,000
8	100,000	60,000	160,000
9	100,000	60,000	160,000
10	100,000	60,000	160,000

The accounting earnings do *not* accurately reflect the actual timing of cash flows.

## 2.2 Use After-tax Cash Flows

**Example.** Consider the following project (the cash flow is in thousands of dollars):

Year	0	1	2	3	4	5
Invest	500					
Operating CF		0	100	300	300	300
Depreciation		100	100	100	100	100
Income		-100	0	200	200	200
Tax		-50	0	100	100	100
After-tax CF	-500	50	100	200	200	200
PV at 10%	-500	45.45	82.64	150.26	136.60	124.18

$$NPV = +39.13.$$

How about the following “argument”:

1. Suppose we want to earn 10% after tax and tax rate is 50%
2. Then, it is OK to earn 20% before tax.

This “argument” is, in general, incorrect: A project earning 20% before 50% tax does not usually earn 10% after tax. It may earn either more or less.

In the above example, the pre-tax return is 20.5% while the after-tax return is 12.5%. (Here, the return on a cash flow is defined as its IRR.)

Accelerated depreciation would increase after-tax return further with the same pre-tax return.

## 2.3 Investment In WC Is A Capital Expenditure

Typically, there are timing differences between the accounting measure of earnings (Sales - Cost of Goods Sold) and cash flows.

$$\text{Working Capital (WC)} = \text{Inventory} + \text{A/R} - \text{A/P}.$$

### Changes in Working Capital

- Inventory: Cost of goods sold includes only the cost of items sold. When inventory is rising, the cost of goods sold understates cash outflows. When inventory is falling, cost of goods sold overstates cash outflows.
- Accounts Receivable (A/R): Accounting sales may reflect sales that have not been paid for. Accounting sales understate cash inflows if the company is receiving payment for sales in past periods.
- Accounts Payable (A/P).

**Example.** You run a chain of stores that sells sweaters. This quarter, you buy 1,000,000 sweaters at a price of \$30.00 each. For the next two quarters, you sell 500,000 sweaters each quarter for \$60.00 each. The corporate tax rate is 40%. In million dollars, your cash flows are

Date	After Tax Operating Profit	Inventory	Cash Flow
0	0	$(1)(30) = 30$	-30
1	$(0.5)(60-30)(1-0.4) = 9$	$(0.5)(30) = 15$	$(0.5)(60) - (0.5)(60-30)(0.4) = 24$
2	$(0.5)(60-30)(1-0.4) = 9$	0	$(0.5)(60) - (0.5)(60-30)(0.4) = 24$

Note:

$$\text{Cash flow} = \text{Profit (after tax)} - \text{Change in Inventory.}$$

**Example** (Gromb). MSW Inc. is considering the introduction of a new product: Turbo-Widgets (TW).

- TW were developed at an R&D cost of \$1M over past 3 years
- New machine to produce TW would cost \$2M
- New machine lasts for 15 years, with salvage value of \$50,000
- New machine can be depreciated linearly to \$0 over 10 years
- TW need to be painted; this can be done using excess capacity of the painting machine, which currently runs at a cost of \$30,000 (regardless of how much it is used)
- Operating cost: \$40,000 per year
- Sales: \$400,000, but cannibalization would lead existing sales of regular widgets to decrease by \$20,000
- Working Capital (WC): \$250,000 needed over the life of the project
- Tax rate: 34%
- Opportunity cost of capital: 10%.

Question: What is the project's NPV (i.e., should MSW go ahead with the production of TW)?

1. Initial investment includes capital expenditure and WC
2. R&D expense is a sunk cost
3. Depreciation is  $\$2\text{M}/10 = \$0.2\text{M}$  for first 10 years
4. Project should not be charged for painting-machine time
5. Project should be charged for cannibalization of regular widget sales
6. Salvage value is fully taxable since the book value at the end of year 10 is  $\$0$  (the machine cost has been fully depreciated).

The cash flows (in thousand dollars) are

Year	Cash Flow
0	$-(2000+250) = -2250$
1-10	$(400-40-20)(1-0.34) + (200)(0.34) = 292.4$
11-14	$(400-40-20)(1-0.34) = 224.4$
15	$224.4 + (50)(1-0.34) + 250 = 507.4$

$$NPV = -\$57,617.$$

### 3 Discount Rates

Recall that, given a project's (expected) cashflows

$$\{CF_0, CF_1, \dots, CF_t\}$$

its NPV is

$$NPV = CF_0 + \frac{CF_1}{1 + r_1} + \dots + \frac{CF_t}{(1 + r_t)^t}.$$

Knowing the cashflows of a project, we need to find the appropriate discount rates to compute its NPV.

We have learned through 15.407 that

- A project's discount rate (i.e., required rate of return) is the expected rate of return demanded by investors for the project.
- The discount rate(s) in general depend on the timing and risk of the cashflow(s).
- The discount rate is usually different for different projects. (Thus, it is in general incorrect to use a company-wide "cost of capital" to discount cash flows of all projects.)

In this section, we apply the pricing theories we have learned, CAPM and APT, to answer the question:

- What is the required rate of return on a project?
  - (a) We first consider the simple case in which a single discount rate can be used for all cashflows of a project (the term structure of discount rates is flat).
  - (b) We then discuss the general case (the term structure of discount rates is not flat).

### 3.1 Using CAPM to Estimate Cost of Capital

CAPM states that a project's required rate of return is determined by the beta of the *project*:

$$\bar{r} = r_F + \beta (\bar{r}_m - r_F)$$

where

- $r_F$  is the risk-free rate
- $\beta$  is the beta of the project with respect to the market
- $\bar{r}_m$  is the required rate of return on the market.

**Example** A publishing house, say McGraw Hill, is considering buying a software company, SuperSoft. How to calculate the “fair” market value of SuperSoft?

- McGraw Hill should not use its own beta to discount SuperSoft's cash flows.
- McGraw Hill should use the beta of SuperSoft's cash flows.

Question: How to estimate SuperSoft's beta?

Case 1: Suppose that SuperSoft has its common stock publicly traded. As discussed before, we can use realized returns on SuperSoft's stock to estimate its beta.

Suppose that  $\hat{\beta} = 1.4$ ,  $\bar{r}_m - r_F = 8.6\%$ , and the current riskless interest rate is 5%.

- Estimated required rate of return on SuperSoft stock is

$$5\% + (1.4)(8.6\%) = 17.04\%.$$

- Estimated "fair" price per share:

$$\text{Price/Share} = \sum_{t=1}^{\infty} \frac{D_t}{(1+0.1704)^t} = \frac{(1+g)D_0}{(0.1704-g)}$$

where  $g$  is the estimated growth rate of the dividend per share.

- Suppose that  $D_0 = 2$  and  $g = 0.10$ . Then

$$\text{Price/Share} = \frac{(2)(1.1)}{0.1704 - 0.10} = \$31.25.$$

Alternative: Use the current market price of SuperSoft!

Case 2: Suppose that SuperSoft is privately held and is 100% equity-financed. Assume no taxes.

Question: How to price SuperSoft's cash flows?

Answer:

- Find a collection of publicly traded software companies that are in a similar business as SuperSoft and are 100% equity financed.
- Compute the betas of these companies.
- Use an average of these companies' betas as an estimate of SuperSoft's beta.

Caveat:

We may not be so lucky as to find comparable companies with no leverage (100% equity financed). Using betas of companies with different leverage is tricky. We will come back to this issue when we discuss financing.

## 3.2 Using APT to Estimate Cost of Capital

We have discussed the application of APT in estimating the cost of capital in Chapter 12. Please review the discussion there.

### 3.3 Discount Rate and Time Horizon

Discount rates over different horizons are, in general, different.

The term structure of discount rates arises from two sources:

1. Term structure of interest rates: The discount rates in absence of risk can be different for sure cashflows at different dates.
2. Term structure of risk premia:
  - (a) The risk of cashflows at different dates is different.
  - (b) The price of risk is different for different dates.

**Example.** Suppose that  $E[\tilde{r}_m] - r_F = 8.6\%$  and  $r_F = 5\%$ . Consider a financial contract that pays two years from now ( $t = 2$ ) the ratio:

$$\frac{\text{S\&P500}_{\text{year2}}}{\text{S\&P500}_{\text{year1}}} = 1 + \tilde{r}_{m2}.$$

At  $t = 1$ , the price of this contract is

$$\frac{1 + E[\tilde{r}_{m2}]}{1 + E[\tilde{r}_{m2}]} = 1.$$

Since the value of this contract at  $t = 1$  (the end of first year) is certain, the value of this contract today is

$$\frac{1}{1.05} = 0.9524 \quad \text{or} \quad \frac{E[\text{CF}_2]}{(1 + r_{F1})(1 + E[\tilde{r}_{m2}])}.$$

**Example.** A firm is investing in an oil exploration project:

- Drilling takes place over the coming year
- At the end of the first year:
  - with probability  $1/3$ , it finds 3 million barrels of oil
  - with probability  $2/3$ , it finds nothing
- Conditional on successful exploration, 3 million barrels of oil will be produced by the end of the second year. (There is no more oil after that.)
- Expected after-tax profit per barrel is \$20
- $r_F = 5\%$
- Industry discount rate of oil production is 20%
- The exploration risk is unsystematic.

**Question:** What is the value of the right to drill for oil?

- Potential value of the cash flows after drilling (at  $t = 1$ ):

$$\left[ \begin{array}{l} 0 \text{ with probability } 2/3. \\ \frac{(20)(3)}{1.2} = 50 \text{ with probability } 1/3. \end{array} \right.$$

- The value of drilling right now (at  $t = 0$ ):

$$\frac{\frac{2}{3}(0) + \frac{1}{3}(50)}{1.05} = \$15.87\text{M.}$$

Is this really correct? Think about the sources of risk.

## 4 Project Interaction

- With the cash flows of a project and the appropriate (timing- and risk-adjusted) discount rates, we can compute its NPV and make a decision on whether or not to take the project.
- However, often we have to decide among a set of possible projects.
- If these projects are independent, we just apply the same rule for a single project to each one of them: Take a project if its NPV is positive.
- If these projects are dependent (e.g., mutually exclusive—accepting one rules out the others), we have to compare their NPVs. This is the situation we consider in this section: the interaction between projects.

**Example.** Optimal Timing of Projects.

When we consider a project, we often face the following choice:

- Should we reject the project
- Undertake the project now, or
- Undertake the project at some date in the future?

Often, it may be better to wait.

**Example.** Potential demand for your product is projected to increase over time. If you start the project early, your competitors will catch up with you faster, by copying your idea. Your opportunity cost of capital is 10%. Denoting by FPV the project's NPV at the time of introduction, we have:

Year to Start	FPV	% Change in FPV	NPV
1	100	–	91
2	120	20	99
3	138	15	104
4	149	8	102

Before year 4, the return to waiting is larger than the opportunity cost of capital, 10%. As long as the growth rate of FPV remains below 10% after year 4, it is best to wait and introduce at the end of year 3.

## 5 Alternatives to NPV

In practice, other investment rules are also used:

- Payback Period
- Internal Rate of Return (IRR)
- Profitability Index (PI)

Firms use these rules because they were used historically and they may have worked (in combination with common sense) in the particular cases encountered by these firms.

These rules sometimes give the same answer as NPV, but in general they do not. We should be aware of their shortcomings and use NPV whenever possible.

The bottom line is:

The NPV rule dominates these alternative rules.
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## 5.1 Payback Period

Definition: Payback period is the minimum  $s$  so that

$$CF_1 + CF_2 + \cdots + CF_s \geq -CF_0 = I_0$$

In words,  $s$  is the minimum length of time such that the sum of cash flows from a project is positive.

### Decision Criterion Using Payback Period

- For independent projects: Accept if  $s$  is less than or equal to some fixed threshold  $t^*$ .
- For mutually exclusive projects: Among all the projects having  $s \leq t^*$ , accept the one that has the minimum payback period.

**Example.** Let  $t^* = 3$ . Consider the two independent projects with the following cash flows (in thousands):

	$CF_0$	$CF_1$	$CF_2$	$CF_3$	$CF_4$	$CF_5$	$CF_6$	$t^*$
Project 1	-100	20	40	30	10	40	60	4
Project 2	-100	10	10	80	5	10	10	3

Decision: Accept Project 2.

## Problems with Payback Period

- It ignores cash flows after the payback period.
- It ignores discounting.

**Example.** (Continued.) Suppose that the appropriate discount rate is a constant 10% per period. Then

$$NPV_1 = 39,315 \quad \text{and} \quad NPV_2 = -7,270.$$

But we accepted project 2 and not project 1!

## Discounted Payback Period

Taking into account appropriate discounting, we have the discounted payback period:

Definition: Discounted payback period is the minimum  $t^*$  so that

$$\frac{CF_1}{1+r} + \frac{CF_2}{(1+r)^2} + \dots + \frac{CF_{t^*}}{(1+r)^{t^*}} \geq -CF_0$$

where  $r$  is the discount rate.

Problem: It still ignores the cash flows after the discounted payback period.

## 5.2 Internal Rate of Return (IRR)

IRR is the number that satisfies

$$I_0 = \frac{CF_1}{(1 + IRR)} + \frac{CF_2}{(1 + IRR)^2} + \dots + \frac{CF_t}{(1 + IRR)^t}$$

### Decision Criterion Using IRR

- For independent projects: Accept a project if its IRR is greater than some fixed  $IRR^*$ , the threshold rate.
- For mutually exclusive projects: Among the projects having IRR's greater than  $IRR^*$ , accept one with the highest IRR.

**Example** Consider the following mutually exclusive projects:

	$CF_0$	$CF_1$	$CF_2$	$CF_3$	$CF_4$	$CF_5$	$CF_6$
Project 1	-100	20	40	30	10	40	60
Project 2	-100	10	10	80	5	10	10

Then,  $IRR_1 = 21\%$  and  $IRR_2 = 7\%$ .

IRR rule leads to the same decisions as NPV if

1. There is only one cash outflow at time 0
2. Only one project is under consideration
3. Opportunity cost of capital is the same for all periods
4. Threshold rate is set equal to opportunity cost of capital.

## Problems with IRR

### 1. Projects of the loan type:

	$CF_0$	$CF_1$
Project 1	-100	120
Project 2	100	-120

The IRR of both projects is 20%.

If actual opportunity cost is 10%:

- IRR says to accept both projects

However,

- Project 1 has a positive NPV only if  $r < 20\%$ .
- Project 2 has a positive NPV only if  $r > 20\%$ .
- Should take project 1 and reject project 2.

### 2. Non-existence of IRR

	$CF_0$	$CF_1$	$CF_2$
Project 1	-105	250	-150
Project 2	105	-250	150

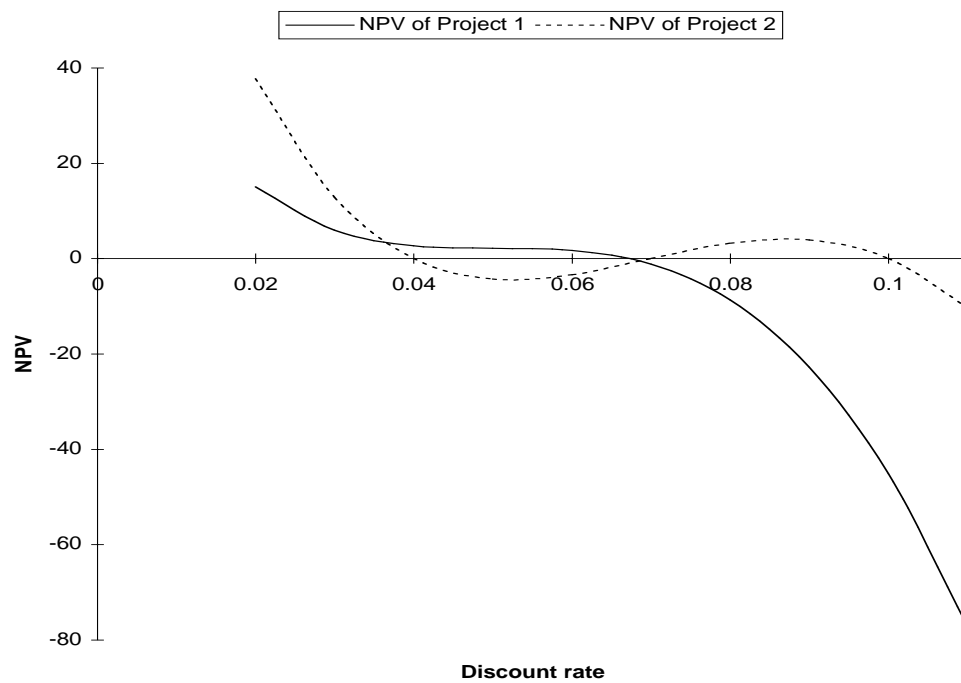
No IRR exists for these two projects.

### 3. Multiple IRR's

	$CF_0$	$CF_1$	$CF_2$	$CF_3$
Project 1	-500,000	1,575,000	-1,653,750	578,815
Project 2	-500,000	1,605,000	-1,716,900	612,040

$$IRR_1 = 7\%$$

$$IRR_2 = \begin{cases} 4\% \\ 7\% \\ 10\% \end{cases}$$



#### 4. Project ranking using IRR for mutually exclusive projects:

##### (a) Projects of different scales:

	$CF_0$	$CF_1$	IRR	NPV at 10%
Project 1	-10,000	20,000	100%	8,181.82
Project 2	-20,000	36,000	80%	12,727.27

A way around this problem is to use *incremental* CF:

- See if lower investment (project 1) is a good idea
- See if incremental investment (project 2) is a good idea.

	$CF_0$	$CF_1$	IRR	NPV at 10%
Project 1	-10,000	20,000	100%	8,181.82
Project 2	-20,000	36,000	80%	12,727.27
Project 2-1	-10,000	16,000	60%	4,545.45

##### (b) Projects with different time patterns of cash flows:

$CF_t$	0	1	2	3	4	5	ETC	IRR	NPV at 10%
Project 1	-90	60	50	40	0	0	...	33%	35.92
Project 2	-90	18	18	18	18	18	...	20%	90.00
Project 2-1	0	-42	-32	-22	18	18	...	15.6%	54.08

## 5.3 Profitability Index

Definition: Profitability Index (PI) is the ratio of the present value of *future* cash flows and the initial cost of a project:

$$PI = \frac{PV}{-CF_0} = \frac{PV}{I_0}.$$

### Decision Criterion Using PI

- For independent projects: Accept all projects with PI greater than one (this is identical to the NPV rule)
- For mutually exclusive projects: Among the projects with PI greater than one, accept the one with the highest PI.

### Problems with PI

PI gives the same answer as NPV when

- (1) There is only one cash outflow, which is at time 0
- (2) Only one project is under consideration.

PI scales projects by their initial investments. The scaling can lead to wrong answers in comparing mutually exclusive projects.

	$CF_0$	$CF_1$	IRR	NPV at 10%	PI at 10%
Project 1	-1,000	2,000	100%	818.18	1.82
Project 2	-2,000	3,600	80%	1,272.73	1.64
Project 2-1	-1,000	1,600	60%	454.55	1.45

## 5.4 The Practice of Capital Budgeting

Comparison of Methods Used by Large U.S. and Multinational Firms

	Large U.S. Firms	Multinationals	
	Percentage Using Each Method	Use as Primary Method	Use as Secondary Method
Payback Period	80.3%	5.0%	37.6%
IRR	65.5	65.3	14.6
NPV	67.6	16.5	30.0
Other	-	2.5	3.2

Historical Comparison of Primary use of Various Capital Budgeting Techniques

	1959	1964	1970	1975	1977	1979	1981
Payback Period	34%	24%	12%	15%	9%	10%	5.0%
IRR	19	38	57	37	54	60	65.3
NPV	-	-	-	26	10	14	16.5
IRR or NPV	19	38	57	63	64	74	81.8

Source: S. Ross, R. Westerfield, and B. Jordon, *Essentials of Corporate Finance*, Irwin, 1996.)

## 6 Other Issues in Capital Budgeting

### 1. Competitive Response

- CF forecasts should take into account responses of competitors.

### 2. Capital Rationing.

### 3. Sources of Positive-NPV Projects

- Short-run competitive advantage (right place at right time)
- Long-run competitive advantage (patent, technology, economies of scale, etc.)
- Noise.

## 7 Appendix: Additional Issues

### 7.1 Consistent Treatment of Inflation

**Example.** Two projects have the following cash flows (in \$ M):

Year	0	1	2	3	4
Project A	-1	0.6	0.5	0.4	0.3
Project B	-10	3.0	4.0	5.0	4.0

**Current evaluation:**

	NPV at $r = 10\%$
Project A	+0.46
Project B	+2.52

Suppose now expected inflation increases by 10%. The cost of capital is increased to 21%:

$$1 + r_{\text{nominal}} = (1 + r_{\text{real}})(1 + i).$$

If future cash flows were not restated, we would have

	NPV at $r = 21\%$
Project A	+0.20
Project B	-0.10

However, Project B is still better after the increase in expected inflation if one is consistent in treating inflation:

1. Discount nominal cash flows with nominal discount rate
2. Discount real cash flows with real discount rate.

## 7.2 Use Appropriate Discount Rates

**Example.** In the previous example, suppose discount rates is set at 20%, not because of inflation, but to offset over-optimism of operating managers.

	NPV at $r = 20\%$
A	+0.22
B	0

This adjustment makes sense only if the amount of over-optimism increases by 10% each year:

Year	1	2	3	4
Bias	10%	21%	33%	46%

General Problem: Failure to correct biases introduced by fighting for resources within the firm.

## 7.3 Use Market Values

**Example.** As part of your expansion plan, you want to buy ABC Inc., which is listed on NASDAQ with total market capitalization of \$10 M. How much should you pay?

$$\text{Maximum Payment for Target Firm} \begin{cases} \text{Market Capitalization} + \text{PV}(\text{synergy}) \\ \text{Your DCF Valuation} + \text{PV}(\text{synergy}) \end{cases}$$

Use the market capitalization to estimate the value of traded assets.

## 8 Homework

### Readings:

- BM Chapters 5, 6, 9.

### Assignment:

- Problem Set 9.