M.I.T.
Sloan School of Management
Prof. Denis Gromb

MIDTERM 2
Section B

Number of students: 140
Mean: 65

Grade Distribution:

Letter grades: The class is not graded on a curve. The following are approximations:

- A: from 70 to 100
- B: from 55 to 70
- C: from 35 to 55
- D: from 20 to 35
- E: below 20
Question 1

1) b and c  2) b and c  3) a  4) a

Question 2

Expected returns:

\[ \mathbb{E}[r_A] = \frac{1}{3} \cdot 10\% + \frac{1}{3} \cdot 8\% + \frac{1}{3} \cdot 6\% = 8\% \]

\[ \mathbb{E}[r_B] = \frac{1}{3} \cdot 10\% + \frac{1}{3} \cdot 4\% + \frac{1}{3} \cdot 7\% = 7\% \]

Variances:

\[ \sigma_A^2 = \mathbb{E}[(r_A - \mathbb{E}[r_A])^2] = \frac{1}{3} \cdot (10\% - 8\%)^2 + \frac{1}{3} \cdot (8\% - 8\%)^2 + \frac{1}{3} \cdot (6\% - 8\%)^2 \]

\[ = 0.0002667 \]

\[ \sigma_B^2 = \mathbb{E}[(r_B - \mathbb{E}[r_B])^2] = \frac{1}{3} \cdot (10\% - 7\%)^2 + \frac{1}{3} \cdot (4\% - 7\%)^2 + \frac{1}{3} \cdot (7\% - 7\%)^2 \]

\[ = 0.0006000 \]

Covariance:

\[ \sigma_{AB} = \mathbb{E}[(r_A - \mathbb{E}[r_A]) (r_B - \mathbb{E}[r_B])] \]

\[ = \frac{1}{3} \cdot (10\% - 8\%) (10\% - 7\%) + \frac{1}{3} \cdot (8\% - 8\%) (4\% - 7\%) + \frac{1}{3} \cdot (6\% - 8\%) (7\% - 7\%) \]

\[ = 0.0002000 \]

b) The portfolio with expected return 7.5% has equal weights on assets A and B because

\[ 7.5\% = \frac{1}{2} \cdot 8\% + \frac{1}{2} \cdot 7\% \]

The variance of its return is thus:

\[ \sigma^2 = \left(\frac{1}{2}\right)^2 \sigma_A^2 + \left(\frac{1}{2}\right)^2 \sigma_B^2 + 2 \cdot \frac{1}{2} \cdot \frac{1}{2} \sigma_{AB} = 0.000317 \]

Compared to asset A, this portfolio’s return has a lower mean and a higher variance. Hence, no investor will want to invest all their wealth in it.

Question 3

a) The value of one share is

\[ P = \frac{EPS_1}{r} = \frac{10}{10\%} = $100 \]

It is also given by

\[ P = \frac{D}{r_q} \]
where \( r_q \), the discount rate for a quarter, is given by \((1 + r_q)^4 = (1 + r)\). Hence,
\[
D = P \cdot \left( (1 + r) \frac{1}{4} - 1 \right) = 100 \cdot \left[ 1.1^{0.25} - 1 \right] = $2.41
\]
XYZ's PVGO is:
\[
PVGO = P - \frac{EPS_1}{r} = $0
\]

b) The value of one share is
\[
P = \frac{1}{(1 + r)^b} \cdot \frac{D}{r_q - g}
\]
Hence
\[
g = r_q - \frac{1}{(1 + r)^b} \cdot \frac{D}{P} = \left( (1 + r) \frac{1}{4} - 1 \right) - \frac{1}{1.15} \cdot \frac{7}{100} = -1.9\%
\]
Shareholders are indifferent.

**Question 4**

a) We note that \( R = 1.1, \ u = 1.5 \) and \( d = 0.5 \) so that \( q = \frac{R - d}{u - d} = 0.6 \).
Debt being senior with respect to equity, we have:
\[
S_{uu} = \frac{112.5M - 40M}{500,000} = $145, \ S_{ud} = S_{dd} = 0
\]
Hence,
\[
S_u = \frac{1}{R} [qS_{uu} + (1 - q)S_{ud}] = $79.09
\]
\[
S_d = \frac{1}{R} [qS_{ud} + (1 - q)S_{dd}] = $0
\]
\[
S = \frac{1}{R} [qS_u + (1 - q)S_d] = $43.14
\]

b) European put's payoffs at maturity: \( P_{uu} = 0, \ P_{ud} = P_{dd} = $50 \)
Hence, the premium is:
\[
P = \frac{1}{R^2} \left[ q^2 P_{uu} + 2q(1 - q)P_{ud} + (1 - q)^2 P_{dd} \right] = $26.45
\]

To find the American put’s premium, we need to determine whether it might be exercised prior to maturity. If the stock price goes up to \( S_u = $79.09 \), the put is out of the money (since \( 50 < 79.09 \)), and so it is not optimal to exercise it. Hence \( p_u = \frac{1}{R} [qP_{uu} + (1 - q)P_{ud}] = $18.18 \). If the stock price goes down to \( S_d = $0 \), exercising the put will never yield a higher payoff and so it is optimal to exercise it (if only to gain the time value of money). Hence, \( p_d = $50 \). Finally, exercising the put at \( t = 0 \) would yield \( 50 - 43.14 = $6.86 \). Hence, it is optimal not to exercise the put initially because
\[
\frac{1}{R} [qP_u + (1 - q)p_d] = $28.10 > $6.86
\]
Hence, \( p = \$28.10 \).

**Question 5**

a) One such portfolio (among many others) is:

- 2 long European calls on one share of XYZ with maturity 2 months and strike $20
- 3 written European calls on one share of XYZ with maturity 2 months and strike $40
- 1 long European call on one share of XYZ with maturity 2 months and strike $80

b) One such portfolio (among many others) is:

- 1 long European call on one share of XYZ with maturity 2 months and strike $20
- 2 written European calls on one share of XYZ with maturity 2 months and strike $40
- 2 long European calls on one share of XYZ with maturity 2 months and strike $50
- 2 written European calls on one share of XYZ with maturity 2 months and strike $60
- 1 long European call on one share of XYZ with maturity 2 months and strike $90

c) ![Graph](image1)

d) ![Graph](image2)