

The Major Formulas and Terms For Portfolio Theory, CAPM

1. Formulas :

1. the mean and variance of return of a portfolio

$$r_p = \sum_i (x_i r_i);$$

$$\sigma_p^2 = \sum_i \sum_j (x_i x_j \sigma_{ij})$$

where σ_{ij} is the covariance between assets i and j.

statistical warm-up:

relationship between covariance and correlation:

$$\sigma_{ij} = \rho_{ij} \sigma_i \sigma_j$$

2. the covariance of asset i with the portfolio

$$\sigma_{ip} = \sum_j (x_j \sigma_{ij})$$

3. effect of diversification: suppose there are N iid stocks, each with a variance of σ^2 , then, by forming a portfolio of the N stocks and assigning weights of 1/N on each stock, the total variance is:

$$\sigma_p^2 = \sigma^2 / N$$

4. market beta

$$\beta_i = \sigma_{im} / \sigma_m^2$$

5. CAPM

$$r - r_f = \beta (r_m - r_f)$$

6. APT

$$r - r_f = b_1 (r_{\text{factor 1}} - r_f) + b_2 (r_{\text{factor 2}} - r_f) + \dots$$

2. Concepts:

1. a well-diversified portfolio has only market risk.

2. an absolutely safe portfolio has zero variance
3. diversification does not create value: what firms can diversify can also be diversified by investors. This leads to the concept of value additivity.
4. the efficient portfolio when there is a risk-free asset: it is the line connecting the risk-free asset and the market portfolio
5. the portfolio of risky asset with the highest attainable ratio of expected risk premium to portfolio standard deviation is the market portfolio

The definition of efficient market

- Weak form efficiency: The current prices reflect all information contained in the record of past prices.
- Semi-strong form efficiency: The current prices reflect not only past prices but all other published information, such as announcement of dividends and earnings, forecasts of company earnings, changes in accounting practices, mergers, etc.
- Strong form efficiency: The current prices reflect not just public information but all the information that can be acquired by painstaking analysis of the company and the economy. There will be no successful investment managers who can consistently beat the market.

Note: How to find easily whether a market is weak, semi-strong, strong form efficient?

- The key point of market efficiency is that we can not earn easy money with the information permitted to use in each form of efficiency.
- Problem 7 of problem set 6

Abnormal return: Realized return - Expected return

$AR_{i,t}$ (Abnormal return of stock i at time t) = $r_{i,t} - E[r_{i,t} | \Omega_{t-1}]$ where Ω_{t-1} is information of time t-1

e.g. In CAPM, $AR_{i,t} = r_{i,t} - [r_f + \beta^*(r_{m,t} - r_f)]$,

In one-factor market model (e.g. Merrill Lynch table), $AR_{i,t} = r_{i,t} - [\alpha_i + \beta^*r_{m,t}]$

- Problem 6 of problem set 6

The Constant-growth Model:

If dividends are expected to grow at a constant rate g, then, $P_0 = \frac{DIV_1}{r_E - g}$ (Provided $g < r_E$)

Conversely, the market capitalization rate r is the dividend yield plus the rate of dividend growth:

$$r_E = \frac{DIV_1}{P_0} + g$$

Weighted-average cost of capital formula

$$WACC = r_D \cdot (1 - t) \cdot \frac{D}{V} + r_E \cdot \frac{E}{V}$$

- Problem 1 of problem set 6

Beta of assets

$$b_A = \left(\frac{D}{D+E} \times b_D\right) + \left(\frac{E}{D+E} \times b_E\right)$$

Miller and Modigliani Theorem:

MM1: The firm value is independent of the proportion of debt to equity.

MM2: The expected rate of return on the common stock of a levered firm increases in proportion to the debt-equity ratio (D/E), expressed in market values. (Must see and understand Figure 17-2)

Problem 2 of problem set 6

Important reminder of cost of capital

We always think the cost of capital as an **opportunity cost**.

Dividend Policy

Without tax or signaling issue, tax or repurchasing decision does not affect the value of a firm.

Comparison between stock repurchasing and dividend - Problem 5 of problem set 6

Key point:

- The company should pay out the same amount of repurchasing payment as they do as a dividend.
- Be careful about the reduced number of outstanding stock after repurchasing when you calculated the revised stock price.