MIT Sloan School of Management

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E52-435

Finance Theory 15.415
Spring 1999

Solution to Assignment 1: Present Value

Due: February 18 (Thursday), 1999

1. NFL Television Rights

(a) The reported amounts do not take into account the time value of money. Thus, the undiscounted reported amounts, are greater than the present value of the promised amounts.

(b) The PV of the eight payments that ABC has to make, with the first payment starting right away, is:

$$\sum_{t=0}^{7} \frac{550}{1.059^t} = \$3.63125\,\text{billion},$$

which is substantially less than the reported \$4.4 billion.

Similarly, the PV of the payments to be made by ESPN is:

$$\sum_{t=0}^{7} \frac{600}{1.059^t} = \$3.96137\,\text{billion},$$

which is a lot smaller than \$4.8 billion.

2. The Reborn VW Beetle

First use the annuity formula to determine the monthly payments C_a and C_b for dealerships A and B, respectively, ignoring the initial down payments:

(a) Dealership A: $\text{PV}_a = \$18{,}000$, $r_a = 0.08/12$, and $t = 36$ months $\Rightarrow C_a = \$564.05$

(b) Dealership B: $\text{PV}_b = \$15{,}500$, $r_b = 0.10/12$, and $t = 36$ months $\Rightarrow C_b = \$500.14$

If the monthly discount rate is currently r, then the net present values of the two packages are

$$\begin{aligned}
\text{NPV}_a &= 2{,}000 + C_a \left[\frac{1}{r} - \frac{1}{r(1+r)^{36}} \right], \\
\text{NPV}_b &= 4{,}000 + C_b \left[\frac{1}{r} - \frac{1}{r(1+r)^{36}} \right].
\end{aligned}$$

It is clearly more advantageous to accept dealership A's offer if and only if $\text{NPV}_a < \text{NPV}_b$. Substituting the expressions from above and simplifying, we have that $\text{NPV}_a < \text{NPV}_b$ if and only if

$$\left[\frac{1}{r} - \frac{1}{r(1+r)^{36}}\right] < 31.29$$

By trial and error, the cross-over point is at $r = 0.00778$. The conclusion is that if the current annual interest rate for a 36-month period (compounded monthly) is above 9.34%, you should choose dealership A. If the current annual interest rate for a 36-month period (compounded monthly) is below 9.34%, you should choose dealership B.

3. Buying a lottery

(a) We get that Singer paid $139,999.86 by solving the equation

$$\text{Buying Price} = \sum_{t=1}^{9} \frac{65276 \times 0.5}{(1+0.18095)^t}.$$

To get the selling price, we solve

$$\text{Selling Price} = \sum_{t=1}^{9} \frac{65276 \times 0.5}{(1+0.0896)^t}.$$

Given that Singer paid $139,999.86 and received $195,993.83, implies a profit of $55,993.97.

(b) No – the statement is false because it ignores the time value of money.

(c) The amount is $50,000 + present value of a 19-year ordinary annuity at 7.16% interest, or $\$50,000 + \$510,636.42 = 560,636.42$.

4. IRA Accounts and Taxes

(a) Yearly contribution: $2000 \times (1 - 28\%) = \$1440$
UW approach:

$$\sum_{t=1}^{30} \left(2000 \times (1-0.28) \times [1 + 0.06 \times (1-0.28)]^{30-t}\right) = \$85,218.$$

VW approach:
Effective interest rate: $6\% \times (1 - 28\%) = 4.32\%$
Note this is a "future value" question. There could be two ways to calculate the future value:
(1) use a future value formula: $\frac{x \times ((1+r)^N - 1)}{r}$
(2) first calculate the NPV of the annuity, then times $(1+r)^N$ to get the future value

I am doing it with the second method. You should be able to verify that the first method gives the same result.

$$1440/4.32\% \times (1 - \frac{1}{(1+4.32\%)^{30}}) \times ((1+4.32\%)^{30}) = \$85,218.$$

(b) UW: To get the total amount at end of 30 years:

$$\sum_{t=1}^{30} \left(2000 \times (1-0.28) \times [1 + .06 \times (1 - .00)]^{30-t}\right) = \$113,843.$$

of which the principal is $\sum_{t=1}^{30} 2000 \times (1-0.28) = \$43,200$, implying that the interest is $113,843 - 43200 = \$70643.79$. Tax on this is 70643.79×0.28 implying that the take-home amount is $94,063.

VW approach:

$$1440/6\% \times (1 - 1/(1+6\%)^{30}) \times ((1+6\%)^{30}) = \$113,843.$$

Tax on interest accrued: $(113843 - 1440 \times 30) \times 0.28 = 19,780$.

Take-home amount: $113843 - 19780 = \$94,063$.

(c) UW approach: The total amount is

$$\sum_{t=1}^{30} \left(2000 \times [1 + .06]^{30-t}\right) = \$158116.37.$$

After taxes, this is only $158116.37 \times (1 - 0.28) = 113,843.789$.

VW approach:

$$\frac{2000}{6\%} \times (1 - \frac{1}{(1+6\%)^{30}}) = \$158,116.$$

$158116 \times (1 - 28\%) = \$113,843$.

(d) Increase, because more deferred tax can be used to accrue interest.