MIT Sloan School of Management

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Finance Theory 15.415
Spring 1999

Solution to Assignment 2: Valuation of Fixed-Income Securities

1. Bond underwriting

If the underwriter purchases the bonds from the corporate client, then it assumes the full risk of being unable to resell the bonds at the stipulated offering price. In other words, the underwriter bears the risk of interest rate movement between the time of purchase and the time of resale. For long maturity bonds, it is generally true that its duration is also long. Recall that $-\mathcal{D} = \frac{\Delta V/V}{\Delta(1+r)/(1+r)}$ Thus, bonds with long maturities are more exposed to interest rate movement risk. Therefore, the underwriter demands a larger spread (higher underwriting fees) between the purchase price and stipulated offering price.

2. Mortgage payments

(a) To get the payments for the fixed-rate mortgage, first find out the nominal interest rate. This is given by:

$$[(1.0385) \times (1 + 0.025) - 1] + 0.001 = 0.0654625.$$

Then, the annual payment is obtained by solving:

$$500000 = \sum_{t=1}^{10} \frac{\text{Annuity}}{(1 + 0.0654625)^t)},$$

which gives: Annuity = \$69,703.

(b) For the floating rate mortgage, see the spreadsheet below.

Fixed vs. Floating Rate Mortgages

Time	Exp Infln	Nom int rate	Fixed pmt	Computations for floating payment			
				Floating pmt	Interest paid	Principal repaid	Principal o/s
0	0.0250	0.064462					500,000
1	0.0255	0.064982	-69,703	-69,378	-32,231	-37,146	462,854
2	0.0260	0.065501	-69,703	-69,533	-30,077	-39,456	423,398
3	0.0265	0.066020	-69,703	-69,674	-27,733	-41,941	381,457
4	0.0270	0.066540	-69,703	-69,802	-25,184	-44,618	336,839
5	0.0275	0.067059	-69,703	-69,914	-22,413	-47,501	289,338
6	0.0280	0.067578	-69,703	-70,012	-19,403	-50,609	238,729
7	0.0285	0.068097	-69,703	-70,094	-16,133	-53,962	184,767
8	0.0290	0.068616	-69,703	-70,161	-12,582	-57,579	127,188
9	0.0295	0.069136	-69,703	-70,212	-8,727	-61,485	65,703
10	NA	NA	-69,703	-70,246	-4,542	-65,703	0

3. IOPO Securities

(a) There reason for stripping bonds to create IO or PO securities is because there is demand from investors for these securities. Investor's often want to hold zero coupons securities because (i) they then do not have to worry about reinvestment risk (taxes on interest income etc.); (ii) a zero coupon bond is an efficient hedging instrument because its duration is always the time to maturity—thus, hedging with it may significantly reduce rebalancing. Note however, that a PO is not exactly a zero coupon bond (it receives the payments towards principal). The PO instruments also allow one to make a play on interest rates, which determine prepayment.

On the other hand, investors who desire a regular income may wish to hold the IO bond.

(b) The duration for the IO and PO securities is computed below. As one would expect, the duration is much higher for the PO security.

Duration Computations

Time	*Discount rate*	*Present value of payments*			*t * PV/TotalPV*		
	using r=.0654625	Fixed pmt	Interest paid	Principal repay	Fixed pmt	Interest paid	Principal repay
0	1.0000						
1	0.9386	-65,420	-30,720	-34,700	0.13	0.20	0.10
2	0.8809	-61,401	-26,701	-34,700	0.25	0.35	0.20
3	0.8268	-57,628	-22,928	-34,700	0.35	0.45	0.30
4	0.7760	-54,088	-19,388	-34,700	0.43	0.51	0.40
5	0.7283	-50,765	-16,064	-34,700	0.51	0.52	0.50
6	0.6836	-47,646	-12,945	-34,700	0.57	0.51	0.60
7	0.6416	-44,718	-10,018	-34,700	0.63	0.46	0.70
8	0.6021	-41,971	-7,271	-34,700	0.67	0.38	0.80
9	0.5651	-39,392	-4,692	-34,700	0.71	0.28	0.90
10	0.5304	-36,972	-2,272	-34,700	0.74	0.15	1.00
		-500,000	**-152,998**	**-347,002**	**4.98**	**3.80**	**5.50**
			Total PV			**Duration**	

4. Inflation-Indexed Bonds

(a) The cashflows of a standard and an indexed-bond are compared below. Note that to get the numbers reported in the newspaper, interest is computed on the current principal rather than that in the last period. Also, inflation is compounded annually by the treasury, even though coupons are paid semi-annually. In our case, we have compounded inflation semi-annually (as done in the HBS case # 298-017: Treasury Inflation-Protected Securities (TIPS)).

Cashflows of Standard Note vs. Indexed Note

Year	Payment Date	STANDARD NOTE Principal	STANDARD NOTE Interest	INDEXED NOTE Principal	INDEXED NOTE Interest		
0.0	1/29/97	$1,000.00		$1,000.00			
0.5	7/29/97	$1,000.00	$30.00	$1,015.00	$15.23	=0.015*1015	
1.0	1/29/98	$1,000.00	$30.00	$1,030.23	$15.45	=0.015*1030.23	
1.5	7/29/98	$1,000.00	$30.00	$1,045.68	$15.69		
2.0	1/29/99	$1,000.00	$30.00	$1,061.36	$15.92		
2.5	7/29/99	$1,000.00	$30.00	$1,077.28	$16.16		
3.0	1/29/00	$1,000.00	$30.00	$1,093.44	$16.40		
3.5	7/29/00	$1,000.00	$30.00	$1,109.84	$16.65		
4.0	1/29/01	$1,000.00	$30.00	$1,126.49	$16.90		
4.5	7/29/01	$1,000.00	$30.00	$1,143.39	$17.15		
5.0	1/29/02	$1,000.00	$30.00	$1,160.54	$17.41		
5.5	7/29/02	$1,000.00	$30.00	$1,177.95	$17.67		
6.0	1/29/03	$1,000.00	$30.00	$1,195.62	$17.93		
6.5	7/29/03	$1,000.00	$30.00	$1,213.55	$18.20		
7.0	1/29/04	$1,000.00	$30.00	$1,231.76	$18.48		
7.5	7/29/04	$1,000.00	$30.00	$1,250.23	$18.75		
8.0	1/29/05	$1,000.00	$30.00	$1,268.99	$19.03		
8.5	7/29/05	$1,000.00	$30.00	$1,288.02	$19.32		
9.0	1/29/06	$1,000.00	$30.00	$1,307.34	$19.61	1000*(1.03)^10	=1343.91
9.5	7/29/06	$1,000.00	$30.00	$1,326.95	$19.90	1000*(1+0.03/2)^20	=1346.86
10.0	1/29/07	$1,000.00	$30.00	$1,346.86	$20.20		
					$352.06	*Total interest*	

(b) The value of the two bonds is computed using prices of zero-coupon bonds, inferred from the prices of treasury bonds. Linear interpolation of discount factors was used to determine the prices for the zeros that are not available.

Compute Price of Cashflows of the Standard and Indexed Notes

Year	STANDARD NOTE Cashflow	STANDARD NOTE Discount Rate	STANDARD NOTE Present Value	INDEXED NOTE Cashflow	INDEXED NOTE Discount Rate	INDEXED NOTE Present Value
0.0						
0.5	$30.00	0.9726	$29.18	$15.23	0.9726	$14.81
1.0	$30.00	0.9434	$28.30	$15.45	0.9434	$14.58
1.5	$30.00	0.9160	$27.48	$15.69	0.9160	$14.37
2.0	$30.00	0.8887	$26.66	$15.92	0.8887	$14.15
2.5	$30.00	0.8609	$25.83	$16.16	0.8609	$13.91
3.0	$30.00	0.8332	$25.00	$16.40	0.8332	$13.67
3.5	$30.00	0.8071	$24.21	$16.65	0.8071	$13.44
4.0	$30.00	0.7810	$23.43	$16.90	0.7810	$13.20
4.5	$30.00	0.7549	$22.65	$17.15	0.7549	$12.95
5.0	$30.00	0.7288	$21.86	$17.41	0.7288	$12.69
5.5	$30.00	0.7054	$21.16	$17.67	0.7054	$12.46
6.0	$30.00	0.6819	$20.46	$17.93	0.6819	$12.23
6.5	$30.00	0.6585	$19.75	$18.20	0.6585	$11.99
7.0	$30.00	0.6350	$19.05	$18.48	0.6350	$11.73
7.5	$30.00	0.6154	$18.46	$18.75	0.6154	$11.54
8.0	$30.00	0.5958	$17.87	$19.03	0.5958	$11.34
8.5	$30.00	0.5762	$17.29	$19.32	0.5762	$11.13
9.0	$30.00	0.5566	$16.70	$19.61	0.5566	$10.91
9.5	$30.00	0.5370	$16.11	$19.90	0.5370	$10.69
10.0	$1,030.00	0.5174	$532.88	$1,367.06	0.5174	$707.26
PV OF NOTE			$954.34			$949.04

(c) Clearly, TIPS are more attractive if inflation is high. If inflation were 3.5% instead of 3%, the price of the index bond would be $990.42 instead of $949.04.

Compute Price of Cashflows of the Standard and Indexed Notes
with inflation = 3.5%

	STANDARD NOTE			INDEXED NOTE		
Year	Cashflow	Discount Rate	Present Value	Cashflow	Discount Rate	Present Value
0.0						
0.5	$30.00	0.9726	$29.18	$15.26	0.9726	$14.84
1.0	$30.00	0.9434	$28.30	$15.53	0.9434	$14.65
1.5	$30.00	0.9160	$27.48	$15.80	0.9160	$14.47
2.0	$30.00	0.8887	$26.66	$16.08	0.8887	$14.29
2.5	$30.00	0.8609	$25.83	$16.36	0.8609	$14.08
3.0	$30.00	0.8332	$25.00	$16.65	0.8332	$13.87
3.5	$30.00	0.8071	$24.21	$16.94	0.8071	$13.67
4.0	$30.00	0.7810	$23.43	$17.23	0.7810	$13.46
4.5	$30.00	0.7549	$22.65	$17.53	0.7549	$13.24
5.0	$30.00	0.7288	$21.86	$17.84	0.7288	$13.00
5.5	$30.00	0.7054	$21.16	$18.15	0.7054	$12.81
6.0	$30.00	0.6819	$20.46	$18.47	0.6819	$12.60
6.5	$30.00	0.6585	$19.75	$18.79	0.6585	$12.38
7.0	$30.00	0.6350	$19.05	$19.12	0.6350	$12.14
7.5	$30.00	0.6154	$18.46	$19.46	0.6154	$11.98
8.0	$30.00	0.5958	$17.87	$19.80	0.5958	$11.80
8.5	$30.00	0.5762	$17.29	$20.15	0.5762	$11.61
9.0	$30.00	0.5566	$16.70	$20.50	0.5566	$11.41
9.5	$30.00	0.5370	$16.11	$20.86	0.5370	$11.20
10.0	$1,030.00	0.5174	$532.88	$1,436.00	0.5174	$742.93
PV OF NOTE			$954.34			$990.42

5. Hedging interest rate risk

(a) Given that the price of Bond A was calculated using a 10% discount rate that is the same as the YTM, we can just consider an investment "I" in bond A that should equal $1 million discounted from year 5.

$$\begin{aligned} \text{Investment} &= I = \frac{1000000}{(1+.1)^5} = 620921 \\ \text{Number of Bond A} &= \frac{I}{Price_A} = \frac{620921}{118.95} = 5220.02 \text{ Bonds} \end{aligned}$$

Given that $B_A = \$118.95$, we need to invest $N \cdot B_A = 5220.02 \cdot 118.95 = 620921$.

(b) Using the Future value formula described below we obtain:

$$\begin{aligned} FV(@9\%) &= N \cdot \left(\frac{C}{r}[(1+r)^5 - 1] + P\right) \\ &= (5220.02)\left(\frac{15}{.09}[(1+.09)^5 - 1] + 100\right) = 990606 \\ FV(@11\%) &= (5220.02) \cdot \left(\frac{15}{.11}[(1+.11)^5 - 1] + 100\right) = 1009640. \end{aligned}$$

$FV(@9\%)$ decreased .94% while the $FV(@11\%)$ increased .96%.

i.

$$\begin{aligned} D_A &= 3.95 \\ D_B &= 6.28 \end{aligned}$$

Choose ω such that $\omega D_A + (1-\omega)D_B = 5$. This happens when $\omega = 0.5494$. This implies that $(.5494)(620,921) = 341134$ should be invested in bond A and the remaining amount ($279,786) in bond B.

ii.

$$\begin{aligned} FV_{portfolio}(@11\%) &= N_A \cdot FV_A(@11\%) + N_B \cdot FV_B(@11\%) \\ \text{where } N_A &= \frac{341,134}{118.95} = 2867.88 \\ \text{and } N_B &= \frac{279,786}{130.72} = 2140.35. \end{aligned}$$

The future value is $1,000,272.

iii. The new durations and prices for the bonds are as follows:

$$\begin{aligned} P_A &= 119.44 \\ P_B &= 135.97 \\ \mathcal{D}_A &= 3.35 \\ \mathcal{D}_B &= 5.98 \end{aligned}$$

Choose ω such that $\omega\mathcal{D}_A + (1-\omega)\mathcal{D}_B = 4$.
This happens when $\omega = 0.7529$. This implies that this fraction of the remaining portfolio value and the coupon payments that were received should be invested in bond A. That is,

$$(0.7529)[(2867.7)(P_A + C_A) + (2140.6)(P_B + C_B)] = 533,578.$$

should be invested in A and the remaining amount ($175,119) in bond B. The future value of this portfolio in 4 years if rates rise to 10% is found by

$$\begin{aligned} FV_{portfolio}(@10\%) &= N_A \cdot FV_A(@10\%) + N_B \cdot FV_B(@10\%) \\ \text{where } N_A &= \frac{533,579}{119.44} = 4467 \\ \text{and } N_B &= \frac{175,119}{135.97} = 1288 \end{aligned}$$

The future value is about $1,000,598.