MIT Sloan School of Management

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Solutions to Assignment 6: Portfolio Theory

Beta Management Company

(a) The standard deviations can be calculated using Excel's STDEV() function.

Stock	Cal. REIT	Brown Group	Vanguard 500
St.Dev (StD)	9.23%	8.17%	4.61%

The individual stocks have almost double the variability of the Vanguard Index 500. So the individual stocks are riskier.

California REIT seems riskier than Brown based on Standard Deviation.

(b) The variability of a portfolio with w in asset 1 and $1-w$ in asset 2 is

$$\sigma_p = \left[w^2\sigma_1^2 + 2w(1-w)\sigma_{1,2} + (1-w)^2\sigma_2^2\right]^{\frac{1}{2}}$$

where σ_1, σ_2 are StDs, and $\sigma_{1,2}$ is the covariance between asset 1 and 2. Using Excel function COVAR(), we can calculate the covariance between Vanguard 500 Index and the two stocks.

Stock	Cal. REIT	Brown Group
Cov(Vanguard, Stock)	0.0003	0.0024

- Variability (StD) of the portfolio (99%Vanguard, 1% Cal. REIT)
  $=[(.99^2)(.0461^2) + 2(.99)(.01)(.0003) + (.01^2)(.0923^2)]^{\frac{1}{2}} = 4.57\%$
- Variability (StD) of the portfolio (99%Vanguard, 1% Brown Group)
  $=[(.99^2)(.0461^2) + 2(.99)(.01)(.0024) + (.01^2)(.0817^2)]^{\frac{1}{2}} = 4.61\%$

Comparing these portfolios, we see that the Brown stock adds more variability to the portfolio. Thus, Brown is riskier.

This answer differs from that in part (a) because a large part of the portfolio's risk is related to the covariance between the individual stock and Vanguard. Since the covariance between Brown's stock and Vanguard is almost 8 times that between Cal. REIT and Vanguard, the portfolio that includes Brown is riskier.

(c) The regression results are obtained using the Excel's LINEST() function. Beta can also be calculated using formula Cov(Stock,Vanguard)/Var(Vanguard).

Stock	Beta	Alpha
Cal. REIT	0.1474	-0.0243
Brown Group	1.1633	-0.0195

This is consistent with part (b)'s results because the lower the beta, the less sensitive the stock will be to market movements. The higher beta stock(Brown) is riskier.

(d) The Brown Group stock should have a higher expected return. The riskiness of a stock should be measured by its covariance with the market, rather than its own variance. As shown in part (b), when adding a small amount of a new asset to a diversified portfolio, the covariance between the new asset and the portfolio, rather than the variance of the asset, matters more to the total risk of the final portfolio. In other words, individual risks can be diversified away in a portfolio. But the market risk has to be held by investors. Stock risk premium should be proportional to the market exposure of the stock.

Remark: In Excel, $\text{COVAR}(X,Y) \equiv \frac{1}{n}\sum(x_i-\mu_x)(y_i-\mu_y)$ is a biased estimator for $\text{Cov}(X,Y)$. To be precise, we should have used $\frac{n}{n-2}\text{COVAR}(X,Y)$ to adjust for losing 2 degrees of freedom in calculating the sample means of X and Y. For the purpose of this course, however, we ignore this issue and simply use COVAR() to calculate covariances. (Note that Excel functions $\text{VAR}(X)$ and $\text{STDEV}(X)$ have already adjusted for unbiasedness. Hence, you will get slightly different results for Excel function $\text{VAR}(X)$ and $\text{COVAR}(X,X)$. Either one is fine for this course.)