MIT Sloan School of Management

J. Wang
E52-435

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Solutions to Assignment 9: Financing Decisions

1. Cost of capital using CAPM

(a) The market betas (relative to the market portfolio) that result from the regression are shown below: (Assignment 7)

Asset	β_E
Corporate Bond	.4084
IBM	.9682
TI	1.9369
APPLE	1.7961
HP	1.8865
DEC	1.6135

(b) The asset betas are found by

$$\beta_A = \beta_D \frac{D}{V} + \beta_E \frac{E}{V}$$

Given debt ratios, we can calculate asset betas using the corporate debt and equity beta from part a).

Asset	β_A
IBM	.6833
TI	1.3928
APPLE	1.7067
HP	1.2967
DEC	1.4291
Average	1.3017

(c) The divisional cost of capital, assuming the division's beta is equal to the average beta of the peer companies is given by

$$\begin{aligned} \bar{r}_{\text{division}} &= r_f + \beta_{\text{avg}}(\bar{r}_m - r_f) \\ \text{with } \beta_{\text{avg}} &= 1.3017 \\ \text{yields } \bar{r}_{\text{division}} &= .1168 \end{aligned}$$

(d) When we use the beta of peer group companies to estimate the cost of capital on a project in a division, we are assuming that the project's cash flows are as risky as the assets of the average peer group company.

(e) With tax, the value of a levered firm is equal to the value of the unlevered firm plus the present value of debt tax shield. Or

$$V_L = V_U + \tau D$$

From market data, we observe $V_L = D + E$. So we can write the unlevered firm value, as defined by its assets, as

$$V_A \equiv V_U = (1-\tau)D + E$$

The unlevered firm can be viewed as a portfolio of its debt and equity. Define $w_E(\tau) \equiv \frac{E}{V_U} = \frac{E}{(1-\tau)D+E}$, then from discussion of the beta of a portfolio, we have

$$\beta_A \equiv \beta_U = (1-w_E)\beta_D + w_E\beta_E = \beta_D + w_E(\tau)(\beta_E - \beta_D)$$

Since $w_E(\tau)$ increases in τ and usually $\beta_E > \beta_D$, β_U increases in τ. Thus, if tax can not be ignored, the estimated asset beta from part b) understates the actual asset beta. Or the calculated required return on asset is too low.
We now calculate the asset beta for each company in the peer group. In question b), debt to asset value for each firm is given, or we know $\frac{D}{D+E}$. We can easily calculate $w_E(\tau)$ given current tax rate $\tau = .34$.

Company	$w_E(\tau)$	$\beta_U \equiv \beta_A$
IBM	.5938	.7408
TI	.7327	1.5283
APPLE	.9565	1.7358
HP	.6953	1.4362
DEC	.8935	1.4852
Average		1.3852

Using CAPM and average β_A, the required return on asset is

$$\bar{r}_A = r_f + \beta_{\text{avg}}(\bar{r}_m - r_f) = .04 + (1.3852)(.059) = .1217$$

To figure out the divisional cost of capital, we then need to take into account the debt capacity of this division and corresponding tax shield. Let X be the net cash flow for the division. Define cost of capital WACC as the rate of return that solves

$$V_L = \frac{(1-\tau)X}{\text{WACC}}$$

Also, we know that

$$V_L = V_U + \tau D = \frac{(1-\tau)X}{r_A} + \tau D$$

Combining the two equations, we have

$$\text{WACC} = \left(1 - \tau\frac{D}{V_L}\right) r_A$$

The cost of capital decreases with debt ratio due to the value of debt tax shield. If we assume the division takes on the average debt ratio of the peer group .2963, then

$$\bar{r}_{\text{division}} = [1 - (.34)(.2963)]\,(.1217) = .1095$$