Solution to 15.415 Midterm Exam

(Section C&D)

Spring 1999

1. Let A be the amount of first deposit, then your total contribution is $A(1+i)^t$ for year $t = 0, 1, \cdots, 9$, where $i = 0.04$ is the inflation rate. At (nominal) discount rate $r = 0.06$, the present value of total contribution is

$$\text{PV (contribution)} = \sum_{t=0}^{9} A \frac{(1+i)^t}{(1+r)^t} = A \frac{1-q^{10}}{1-q} \quad \text{where} \quad q \equiv \frac{1+i}{1+r} = 0.9811.$$

   Similarly, the present value of future tuition payments is

$$\text{PV (tuition)} = \sum_{t=10}^{13} (30,000) \frac{(1+i)^t}{(1+r)^t} = (30,000)\ q^{10} \frac{1-q^4}{1-q}$$

   Setting

$$\text{PV (contribution)} = \text{PV (tuition)}$$

   we obtain $A = \$10,488.71$ which is the first payment.

2. (a) Yes, the company is a growth stock because ROE $= 10\%$ is higher than the cost of capital $r = 7\%$.

   (b) $g = b \times \text{ROE} = (0.5)(0.1) = 0.05$ and $D_0 = \$2$. Thus,

$$P_0 = \frac{D_1}{r-g} = \frac{D_0(1+g)}{r-g} = \frac{(2)(1.05)}{0.07-0.05} = 105.$$

   (c) If $g = 0$, then current dividend D_0 becomes to $\text{EPS}_0 = 2/0.5 = \$4$.

$$P_0 = \frac{D_1}{r-g} = \frac{D_0}{r} = \frac{4}{0.07} = 57.14.$$

   Thus, PVGO $= 105 - 57.14 = \$47.86$, or 45.58% of total stock value.

   (d) If payout ratio increases to 60%, then $b = 40\%$, $D_0 = (1-b) \times \text{EPS} = (0.6)(4) = \$2.4$, $g = b \times \text{ROE} = (0.4)(0.1) = 0.04$.

$$P_0 = \frac{D_0(1+g)}{r-g} = \frac{(2.4)(1.04)}{0.07-0.04} = 83.2.$$

   Price drops from 105 to 83.2.

   As a corporate raider, you can buy up the company and decrease payout ratio to 50% to improve the firm value back to 105. (Note that in general, as you increase plow back rate, ROE decreases eventually, because profitable growth opportunities are limited. So after a certain level, you can't increase firm value by increasing plow back. In this problem, we implicitly assumed that the current management forgo profitable investment opportunities by increasing payout.)

3. (a) You should enter 100 1-year futures contracts to buy wheat at \$31 per bushel 1 year from now.

   (b) You gain $(31.20 - 31)(500,000) = \$100,000$.

   (c) Your total position is to pay \$31 a bushel to buy $500,000$ bushels of wheat in year 1, which exactly covers your need of wheat. The net worth is a fixed payment of \$15.5M, independent of future changes in the futures price. Another way to see this is that although you make money on your futures position, you lose the same amount on your short position in wheat (in present value terms). So you total net worth has not changed as the futures price change.

   (d) Let the effective convenience yield be $\widehat{y}$. Then, futures price for maturity T is

   $$F_T = S_0\left(1 + r - \widehat{y}\right)^T.$$

   We know that $r = 0.05$, $S_0 = \$30$, and $F_1 = \$31.2$ for $T = 1$. Thus, $\widehat{y} = 0.01$. The 2-year forward price is then

   $$F_2 = (30)(1.04)^2 = \$32.45.$$

   Note that here, we have assumed that both interest rates and convenience yields have a flat term structure.

4. (a) Solve the following system of equations for the spot rates:

   $$\begin{aligned}
B_A &= \frac{1080}{1+r_1} = 1000 \\
B_B &= \frac{80}{1+r_1} + \frac{1080}{(1+r_2)^2} = 1000 \\
B_C &= \frac{80}{1+r_1} + \frac{80}{(1+r_2)^2} + \frac{1080}{(1+r_3)^3} = 1000
\end{aligned}$$

   which gives $r_1 = r_2 = r_3 = 8\% \equiv y$. Thus the yield curve is flat at 8%, at least for maturities no greater than three years.

   (b) Cash flow is \$1M each for $t = 1, 2, 3$. Hence,

   $$PV = \sum_{t=1}^{3} \frac{1}{(1+y)^t} = 2.5771.$$

   (c) $$D = \left[\sum_{t=1}^{3} \frac{t}{(1+y)^t}\right] / PV = 1.9487.$$

(d) $MD = D/(1+y) = 1.8044$. So when the yield curve shifts up by 0.1%, the PV of the lease income changes by

$$\frac{\Delta V}{V} = -MD(\Delta y) = -(1.8044)(0.001) = -0.018$$

i.e., decreases by 0.18%.

(e) A zero-coupon bond has duration equal to its maturity. So, the zero-coupon bond to be issued should have a maturity of 1.9487 years. The face value P (principal) is chosen such that the present value of the bond equals the present value of lease income, which is 2.5771. Thus

$$P = (2.5771)(1+0.08)^{1.9487} = \$2.9941\text{M}.$$

(f) By matching the duration of asset (future lease income) and liability (promised bond payment), the company is hedged against small parallel shifts in the yield curve. However, because the asset and liability have different convexity, the company is still exposed to interest rate risk due to large movements (or non-parallel shifts) in the yield curve.