Topic 2

NPV VS. OTHER RULES

ROADMAP

Part 0) PRESENT VALUE

Aim: Given expected cash flows and discount rates, find PV

- Topic 1: Definition of PV
- **Topic 2: NPV vs. Other Rules**

Part I) VALUING SECURITIES

Aim: Given expected cash flows and discount rates, price specific securities

Part II) THE PRICING OF RISK

Aim: Determining the risk-adjusted discount rate

Part III) CORPORATE FINANCE

Aim: Study the value implications of financial decisions by firms

Readings: BM 5

OVERVIEW

- Net Present Value (NPV)
- Alternatives:
  - Payback Period (PP)
  - Discounted Payback Period (DPP)
  - Internal Rate of Return (IRR)
  - Profitability Index (PI)

THE NPV RULE

We have defined the Net Present Value of an investment as:

$$-I_0 + \frac{C_1}{(1+r_1)} + ... + \frac{C_n}{(1+r_n)^n}$$

In the stylized world we describe:

- There are no positive NPV deals on the financial markets (no arbitrage opportunities)
- However, there may be positive NPV projects

Firms and individuals have to decide:

- Whether a particular project should be undertaken
- Which of several incompatible projects should be undertaken

The NPV rule is useful for both cases:

- **For independent projects:** Undertake all positive NPV projects
- **For mutually exclusive projects:** Among the positive NPV projects, choose the one with the highest NPV.

UNANIMITY

ALTERNATIVES

- We now examine some alternatives used in practice
  - Payback Period
  - Discounted Payback Period
  - Internal Rate of Return
  - Profitability Index
- We show why they are not as good as the NPV

PAYBACK PERIOD

Payback Period (PP): The minimum period of time needed to recoup the initial investment

It is the smallest T such that:

$$\sum_{t=1}^{T} C_t \geq I_0$$

Some firms (and individuals) make their investment decisions as follows:

- They decide on a **hurdle** (for example: 3 years)
- **For independent projects:** Accept all projects with PP≤ 3
- **For mutually exclusive projects:** Among all projects with PP≤ 3, undertake the one with the shortest PP

IN-CLASS EXERCISE

You are managing one of XYZ Inc.'s divisions. The appropriate discount rate for the following projects is constant at $r = 10\%$

	$-I_0$	C_1	C_2	C_3	C_4	NPV	PP
Proj. A	-100	108	0	0	0	-1.82	
Proj. B	-100	0	120	120	0	89.33	
Proj. C	-100	60	60	120	0	94.29	
Proj. D	-100	0	0	200	-220	-100	
Proj. E	-100	0	0	0	300	104.90	
Proj. F	-200	0	0	0	600	209.81	

XYZ uses a PP rule with a hurdle $T^* = 3$

(a) If the projects are independent which would be accepted?

(b) If the projects are mutually exclusive which would be accepted?

Note: The ranking is independent of the hurdle

SHORTCOMINGS?

We illustrate these shortcomings and point out some possible improvements.

PP SHORTCOMING (1): TIME VALUE OF MONEY?

The payback period rule ignores the time value of money.

This can lead to:

- Undertaking negative NPV projects
- Wrongly ranking mutually exclusive projects

Example: $T^* = 3$ and Project A is available.

$r = 10\%$	$C_0 = -I_0$	C_1	C_2	C_3	C_4	NPV	PP
Proj. A	-100	108	0	0	0	-1.82	1

Project A is undertaken whereas it shouldn't

Example: $T^* = 3$ and Projects B and C are available but mutually exclusive.

$r = 10\%$	$-I_0$	C_1	C_2	C_3	C_4	NPV	PP
Proj. B	-100	0	120	120	0	89.33	2
Proj. C	-100	60	60	120	0	94.29	2

Cannot decide between Projects B and C even though Project C clearly dominates.

DISCOUNTED PAYBACK PERIOD

Some firms use a partial remedy:

Discounted Payback Period: Minimum period of time needed for discounted cash flows to cover the initial investment

It is the smallest T such that:

$$\sum_{t=1}^{T} \frac{C_t}{(1+r_t)^t} \geq I_0$$

Example:

$r = 10\%$	$-I_0$	C_1	C_2	C_3	C_4	NPV	DPP
Proj. A	-100	108	0	0	0	-1.82	
Proj. B	-100	0	120	120	0	89.33	
Proj. C	-100	60	60	120	0	94.29	
Proj. D	-100	0	0	200	-220	-100	
Proj. E	-100	0	0	0	300	104.90	
Proj. F	-200	0	0	0	600	209.81	

PP SHORTCOMING (2): BEYOND THE PP?

The discounted payback rule ignores cash flows after the payback period.

This can lead to:

- Investing in negative NPV projects
- Rejecting positive NPV projects

Example: $T^* = 3$ and Project D is available

$r = 10\%$	$-I_0$	C_1	C_2	C_3	C_4	NPV	DPP
Proj. D	-100	0	0	200	-220	-100	3

Example: $T^* = 3$ and Project E is available

$r = 10\%$	$-I_0$	C_1	C_2	C_3	C_4	NPV	DPP
Proj. E	-100	0	0	0	300	104.90	4

For mutually exclusive projects, the DPP rule ignores cash flows after the smallest DPP.

This can lead to wrongly ranking projects.

Example: $T^* = 5$ and Project B and E are available

$r = 10\%$	$-I_0$	C_1	C_2	C_3	C_4	NPV	DPP
Proj. B	-100	0	120	120	0	89.33	3
Proj. E	-100	0	0	0	300	104.90	4

PP SHORTCOMING (3): SCALE?

The discounted payback rule is insensitive to scale

This can lead to wrongly ranking mutually exclusive projects.

Example: $T^* = 5$ and Projects E and F are available

$r = 10\%$	$-I_0$	C_1	C_2	C_3	C_4	NPV	DPP
Proj. E	-100	0	0	0	300	104.90	4
Proj. F	-200	0	0	0	600	209.81	4

Note: This does not mean that the DPP rule always yields the wrong decision.

INTERNAL RATE OF RETURN

Internal Rate of Return (IRR): The constant discount rate for which the project's NPV **would be** zero

For a project generating cash flows until date n, it is the solution to

$$\sum_{t=1}^{n} \frac{C_t}{(1+IRR)^t} = I_0$$

Note: The IRR is an "abstract" construct unrelated to the actual discount rates, let alone interest rates.

Some firms use the IRR as follows:

- They decide on a threshold IRR^*
- **For independent projects:** Accept all projects with $IRR > IRR^*$
- **For mutually exclusive projects:** Among all projects with $IRR > IRR^*$, choose the one with the highest IRR

EXAMPLE

Note: Except for very simple cases, one cannot solve for the IRR without a calculator or a spread sheet

	$-I_0$	C_1	C_2	C_3	C_4	IRR
Proj. A	-100	108	0	0	0	8%
Proj. B	-100	20	40	30	10	0%
Proj. C	-100	10	10	80	5	1.78%

IRR $\Leftrightarrow$ NPV
IN A SPECIAL CASE

The IRR rule is equivalent to the NPV rule if:

1. Only one project is under consideration
2. All future cash flows are positive (i.e. for $t \geq 1$)
3. The same discount rate is appropriate for all future cash flows and the threshold IRR^* equals that discount rate

Indeed: Consider the function

$$f(x) = -I_0 + \frac{C_1}{1+x} + \frac{C_2}{(1+x)^2} + ... + \frac{C_n}{(1+x)^n}$$

By definition of the IRR: $\quad f(IRR) = 0$

Under (3), $\quad f(IRR^*) = NPV$

Under (2), $\quad f$ is a decreasing function for $x > 0$

Hence, $\quad IRR > IRR^* \Leftrightarrow f(IRR) < f(IRR^*)$

That is: $\quad 0 < NPV$

THE IRR AS A RULE OF THUMB

Here is a way to "justify" the use of the IRR rule to evaluate **one** project.

Faced with the investment decision, one might use the following approximations:

- Future cash flows are risk-free
- The discount rate for safe cash-flows is constant

FEATURES OF IRR

The IRR is widely used. More so than the NPV. Why?

- Concise statistic
- Independent of the estimate of the appropriate discount rate
- Standards die hard

However, prudence is necessary...

IRR SHORTCOMINGS (1)
NEGATIVE CASH FLOWS?

The IRR rule is generally incorrect if some future cash flows are negative.

Example: You consider borrowing $\$100,000$ from me today against a $\$120,000$ repayment next year.

	C_0	C_1	IRR
Loan	100	-120	20%

Assume that the appropriate discount rate is 10%.

- The Loan's NPV is negative (for you!)
- The Loan's $IRR > 10\%$
- Hence, the IRR rule leads you to accept a criminally high interest rate!

Assume that the appropriate discount rate is 25%.

- The Loan's NPV is positive (for you)
- The Loan's $IRR < 25\%$
- The IRR rule leads you to refuse a bargain

IRR SHORTCOMINGS (2)
MULTIPLE IRR?

A cash flow stream can have several IRRs.

Example: $IRR_1 = 5\%$ and $IRR_2 = \begin{cases} 1.588\% \\ 18.75\% \end{cases}$

	$-I_0$	C_1	C_2	C_3	C_4	C_5
Proj. 1	-50	18	5	5	15	15
Proj. 2	-105	50	53	100	100	-200

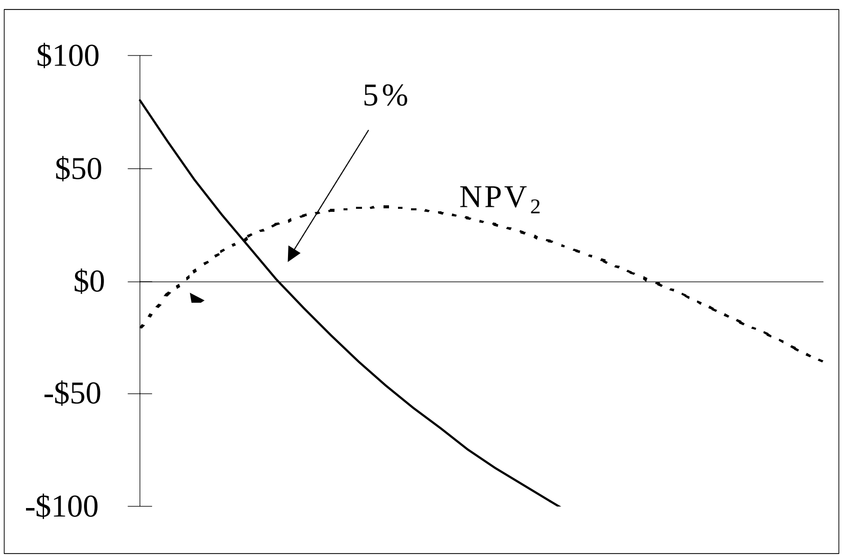

Figure 1: Multiple IRR

Note: The IRR is unique if all future cash flows are positive. Remember that f is decreasing. Hence, it crosses zero only once.

IRR SHORTCOMINGS (3)
EXISTENCE?

A cash flow stream can have no IRR.

Example:

	C_0	C_1	C_2
Proj. 1	-105	250	-150
Proj. 2	105	-250	150

No IRR exists for these two projects.

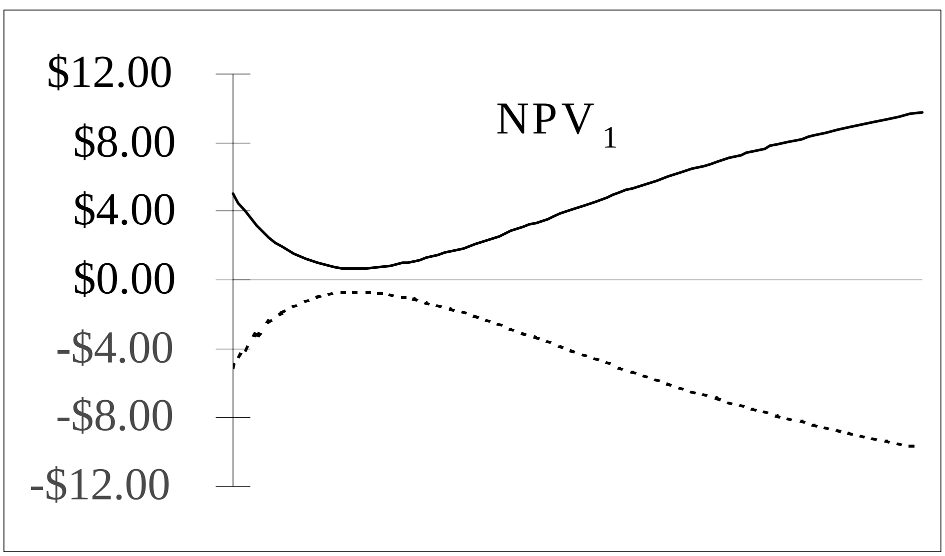

Figure 2: Non Existence of IRR

IRR SHORTCOMINGS (4)
PROJECT SCALE?

The IRR is insensitive to scale.

Example: Assume that the following projects all have an appropriate discount rate of 10%.

	C_0	C_1	IRR	NPV
Proj. 1	-10	20	100%	8.18
Proj. 2	-100	200	100%	81.82
Proj. 3	-100	190	90%	72.73

If you had to choose between Projects 1 and 3, the IRR rule would yield a (very!) wrong decision.

INCREMENTAL CASH FLOWS

A way around this problem:

- See if the project with the lowest investment outlay (Project 1) satisfies $IRR > IRR^*$
- See if the **Incremental Investment** (Project 3 minus 1) satisfies $IRR > IRR^*$

	C_0	C_1	IRR	NPV
Proj. 1	-10	20	100%	8.18
Proj. 3	-100	190	90%	72.73
Proj. (3 -1)	-90	170	89%	64.55

Considering incremental projects:

- Solves the problem
- Boils down to the NPV rule in a complicated way

Note: With more than two projects:

1. See if the project with the lowest investment outlay satisfies $IRR > IRR^*$
2. Consider the project with the next smallest investment outlay and see if the incremental investment satisfies $IRR > IRR^*$
3. Repeat Step 2.

Example: Choosing between Projects A, B and C with appropriate discount rate 10%

	C_0	C_1	IRR	NPV
Proj. A	-10	20	100%	8.18
Proj. B	-100	190		72.73
Proj. C	-200	295		68.18
Proj. (B - A)	-90	170	89%	
Proj. (C - B)	-100	105	5%	

IRR SHORTCOMINGS (5)
DIFFERENT PATTERNS?

The IRR does a poor job at comparing cash flow streams with very different patterns

Example: Assume that the following projects all have an appropriate discount rate of 10%.

	C_0	C_1	C_2	C_3	C_4	..	IRR	NPV
Pr. 1	-90	60	50	40	0	..	33%	36
Pr. 2	-90	18	18	18	18	..	20%	90
Pr. 2-1	0	-42	-32	-22	18	..	15.6%	54

Again, considering incremental projects:

- Solves the problem
- Boils down to the NPV rule in a complicated way

PROFITABILITY INDEX

Profitability Index (PI): Ratio of PV of future cash flows over the initial investment

$$PI = \frac{PV}{-C_0} = \frac{\sum_{t=1}^{n} \frac{C_t}{(1+r_t)^t}}{I_0}$$

Decision criterion using Profitability Index:

- **For independent projects:** Accept all projects with $PI > 1$ (this is identical to the NPV rule)
- **For mutually exclusive projects:** Among all projects with $PI > 1$, accept the one with the highest PI

Example:

$r = 10\%$	C_0	C_1	PI	NPV
Proj. 1	-10	20	1.818	8.18
Proj. 2	-105	110	$.95$	-5

PI SHORTCOMING: SCALE

PI is insensitive to scale

Example:

$r = 10\%$	C_0	C_1	PI	NPV
Proj. 1	-10	20	1.818	8.18
Proj. 2	-100	190	1.73	72.73
Proj. (2 - 1)	-90	170	1.72	64.55

Again, considering incremental projects:

- Solves the problem
- Boils down to the NPV rule in a complicated way

WHAT DO FIRMS DO?

	Large U.S. Firms % Using Each Method	Multinational Primary Method	Multinational Secondary Method
Payback Period	80.3	5.0%	37.6%
Accounting Rate of Return	59.0	10.7	14.6
Internal Rate of Return	65.5	65.3	14.6
Net Present Value (NPV)	67.6	16.5	30.0
Other	-	2.5	3.2

	1959	1964	1970	1975	1977	1979	1981
Payback Period	34%	24%	12%	15%	9%	10%	5.0%
Accounting Rate of Return (ARR)	34	30	26	10	25	14	10.7
Internal Rate of Return (IRR)	19	38	57	37	54	60	65.3
Net Present Value (NPV)	-	-	-	26	10	14	16.5
IRR or NPV	19	38	57	63	64	74	81.8

SPREADSHEET: CAVEAT FOR NPV

To Calculate NPV (with a Constant Discount Rate):

`=npv(rate,range)`

Example: `=npv(10%,A1:D1)`

Warning: The function discounts the first cash flow. So the range should not include the first cash flow.

You can use

`=npv(rate,range)+first cash flow`

Example: `=npv(10%,B1:D1)+A1`

Alternatively, you can use:

`= (1+rate)*npv(rate,range)`

Example: `=(1+10%)*npv(10%,A1:D1)`

SPREADSHEET: CAVEAT FOR IRR

To Calculate IRR

`=irr(range)`

Warning: The function does not indicate multiple IRR's.

If you can guess the approximate IRR, you can use:

`=irr(range,guess)`

Example:

	$-I_0$	C_1	C_2	C_3	C_4	C_5
Proj. 1	-105	50	53	100	100	-200

You would find the two IRRs 1.588% and 18.75% by computing:

`=irr(A1:F1,2%)`

and

`=irr(A1:F1,20%)`