Practice Problems With Solutions For Topic 4

Problems: From Bodie Kane Marcus (various)

BKM 15.1

A nine-year bond has a yield of 10% and a duration of 7.194 years. If the bond's yield changes by 50 basis points, what is the percentage change in the bond's price?

BKM 15.2

Find the duration of a 6% coupon bond making *annual* coupon payments if it has three years until maturity and a yield to maturity of 6%. What is the duration if the yield to maturity is 10%?

3. A pension plan is obligated to make disbursements of $\$1$ million, $\$2$ million and $\$1$ million at the end of each of the next three years, respectively. Find the durations of the plan's obligations if the interest rate is 10% annually.

4. If the plan in question 3 wants to fully fund and immunize its position, how much of its portfolio should it allocate to one-year zero-coupon bonds and perpetuities, respectively, if these are the only two assets funding the plan?

5. You own a fixed income asset with a duration of 5 years. If the level of interest rates, which is currently 8%, goes down by 10 basis points,by how much do you expect the price of the asset to go up (in percentage terms)?

BKM 15.6

Rank the following bonds in terms of ascending duration.

Bond	Coupon	Time to Maturity	Yield to Maturity
A	15%	20 years	10%
B	15%	15	10%
C	0%	20	10%
D	8%	20	10%
E	15%	15	15%

BKM 15.10

You will be paying $\$10,000$ a year in tuition expenses at the end of the next two years. Bonds currently yield 8%.

a. What is the present value and duration of your obligation?

b. What maturity zero-coupon bond would immunize your obligation?

c. Suppose you buy a zero-coupon with value and duration equal to your obligation. Now suppose that rates immediately increase to 9%. What happens to your net position, that is, to the difference between the value of the bond and that of your tuition obligation? What if rates fall to 7%?

BKM 15.14

You are managing a portfolio of $\$1$ million. Your target duration is 10 years, and you can choose from two bonds: a zero-coupon bond with maturity 5 years, and a perpetuity, each currently yielding 5%.

a. How much of each bond will you hold in your portfolio?

b. How will these fractions change *next year* if target duration is now nine years?

12. You manage a pension fund that will provide retired workers with lifetime annuities. You determine that the payouts of the fund are essentially going to resemble level perpetuities of $\$1$ million per year. The interest

rate is 10%. You plan to fully fund the obligation using 5-year maturity and 20-year maturity zero-coupon bonds.

a. How much *market value* of each of the zeros will be necessary to fund the plan if you desire an immunized position?

b. What must be the *face value* of the two zeros to fund the plan?

13. Your client is concerned about the apparent inconsistency between the following two statements.

: Short-term interest rates are more volatile than long-term rates.

: The rates of return of long-term bonds are more volatile than returns on short term securities.

Discuss why these two statements are not necessarily inconsistent.

14. The following questions appeared in past CFA Examinations.

(1) Which set of conditions will result in a bond with the greatest volatility?

a. A high coupon and a short maturity.

b. A high coupon and a long maturity.

c. A low coupon and a short maturity.

d. A low coupon and a long maturity.

(2) An investor who expects declining interest rates would be likely to purchase a bond that has a ... coupon and a ... term to maturity.

a. Low, long.

b. High, short.

c. High, long.

d. Zero, long.

(3) With a zero-coupon bond:

a. Duration equals the weighted average term to maturity.

b. Term to maturity equals duration.

c. Weighted average term to maturity equals the term to maturity.

d. All of the above.

(4) As compared with bonds selling at par, deep discount bonds will have:

a. Greater reinvestment risk.

b. Greater price volatility.

c. Less call protection.

d. None of the above.

Solutions:

BKM 15.1

$$\frac{\Delta P}{P} = -DX\left[\frac{\Delta(1+Y)}{1+Y}\right]$$

$$-7.194X\frac{.5\%}{1.10} = -3.27\%$$

BKM 15.2

$YTM = 6\%$.

(1) Yrs	(2) Pmt.	(3) PV of Pmt	(4) Wt of Pmt	(5) (1) × (4)
1	\$60	\$56.60	0.0566	0.0566
2	\$60	\$53.40	0.0534	0.1068
3	\$1060	\$890.00	0.8900	2.6700
		\$1000.00	1.0000	2.8332

Duration = 2.8332 years.

(1) Yrs	(2) Pmt.	(3) PV of Pmt	(4) Wt of Pmt	(5) (1) × (4)
1	\$60	\$54.55	0.0606	0.0606
2	\$60	\$49.59	0.0551	0.1102
3	\$1060	\$796.39	0.8844	2.6532
		\$900.53	1.0000	2.8240

Duration = 2.824 years, which is less than duration at YTM of 6%.

3.

(1) Yrs	(2) Pmt.	(3) PV of Pmt	(4) Wt of Pmt	(5) (1) × (4)
1	\$1 M	\$0.9091 M	0.2744	0.2744
2	\$2 M	\$1.6529 M	0.4989	0.9978
3	\$1 M	\$0.7513 M	0.2267	0.6801
		\$3.3133 M	1.0000	1.9523

Duration = 1.9523 years.

4.

Let x = wt of zeros;

$1 - x$ = wt of perpetuities.

$$1x + (1-x)\frac{1.10}{0.10} = 1.9523$$

and so $x = 0.9048$.

Place \$3.3133 M $\times 0.9048$ = \$2.9979 M in zeros, and the remainder in perpetuities.

5.

$$
\begin{aligned}
D* &= \frac{D}{1+y} = \frac{5}{1.08} = 4.63 \\
\frac{\Delta P}{P} &= -D * \Delta Y \\
\frac{\Delta P}{P} &= -4.63\,(-0.1\%) = 0.463\%
\end{aligned}
$$

BKM 15.6

C, D, A, B, E.

BKM 15.10

a. PV of obligation:

$$PV = \sum_{t=1}^{2} \frac{\$10,000}{(1.08)^t} = \$17,832.65$$

Duration of obligation:

(1)	(2)	(3)	(4)	(5)
Yr	Pmt.	PV of Pmt	Wt of Pmt	(1) × (4)
1	$\$10,000$	$\$9,259.26$	0.51923	0.51923
2	$\$10,000$	$\$8,573.39$	0.48077	0.96154
		$\$17,832.65$	1.0000	1.48077

Duration of obligation is 1.4808 years.

b. Zero coupon bond with a duration of 1.4808 years would immunize the obligation.

The present value of this bond must be $\$17,832.65$, thus the face value (feature redemption value) must be:

$$\$17,832.65 \times (1.08)^{1.4808} = \$19,985.26$$

c. If interest rates increase to 9%, the value of the bond would be:

$$\frac{\$19,985.26}{(1.09)^{1.4808}} = \$17,590.92$$

The tuition obligation would be:

$$PV = \sum_{t=1}^{2} \frac{\$10,000}{(1.09)^2} = \$17,591.11$$

The net position changes by only $\$0.19$.

If interest rates decline to 7% :

The value of the zero coupon bond would increase to:

$$\frac{\$19,985.26}{(1.07)^{1.4808}} = \$18,079.99$$

The tuition obligation would increase to

$$PV = \sum_{t=1}^{2} \frac{\$10,000}{(1.07)^2} = \$18,080.18$$

The net position changes by $\$0.19$.

As interest rates change, so does the duration of the stream of tuition payments, thus the slight net differences.

BKM 15.14

a. Let $x =$ wt of zero coupon bond and $1 - x =$ wt of perpetuities. Then duration of perpetuity is given by

$$\begin{aligned} \frac{1+y}{y} &= \frac{1.05}{0.05} = 21 \text{ years.} \\ 10 &= 5x + (1-x)\,21 = 0.6875 \end{aligned}$$

Thus $\$1$ M $(0.6875) = \$0.6875$ M in zeros,

and $\$1$ M $(0.3125) = \$0.3125$ M in perpetuities.

b.

$$9 = 4x + (1-x)\,21 = 0.7059$$

Thus $\$1$ M $(0.7059) = \$0.7059$ M in zeros,

and $\$1$ M $(0.2941) = \$0.2941$ M in perpetuities.

12.

a. The present value of the annuities is $\$1$ M $/0.1 = \$10$ M.

The duration is $1.10/0.10 = 11$ years.

Let $x =$ weight of 5 year zeros, and

$1 - x =$ weight of 20 year zeros. Then

$$11 = 5x + 20\,(1-x)$$

and so $x = 0.60$ (in 5 year zeros), and

$1 - x = 0.40$ (in 20 year zeros).

5 year zeros: $\$10$ M $\times 0.60 = \$6$ M market value;

20 year zeros: $\$10$ M $\times 0.40 = \$4$ M market value.

b. Face value of 5 year zeros: $\$6$ M $\times (1.10)^5 = \$9.66$ M;

Face value of 20 year zeros: $\$4$ M $\times (1.10)^{20} = \$26.91$ M.

13.

Although short term rates are more volatile than long term rates, duration effects a bond's price sensitivity to interest rate changes and bonds with longer durations are more price sensitive to interest rate changes. Because short term rates are more volatile it does not mean that short term bond prices are more volatile.

14. **(1)** d; **(2)** d; **(3)** d ;**(4)** b