

Practice Problems With Solutions For Topic 5

Problems: Bodie Kane Marcus: 14.1, 14.2, 14.3, 14.4, 14.5, 14.6, 14.7, 14.9, 14.14

BKM 14.1

- Briefly explain why bonds of different maturities have different yields in terms of the (1) expectations, (2) liquidity, and (3) segmentation hypotheses.
- Briefly describe the implications of each of the three hypotheses when the yield curve is (1) upward sloping, and (2) downward sloping.

BKM 14.2

Which one of the following is false?

- The liquidity preference hypothesis indicates that, all things being equal, longer maturities will have a higher yield.
- The basic conclusion of the expectations hypothesis is that the long-term rate is equal to the anticipated short-term rate.
- The expectations hypothesis indicates a flat yield curve if anticipated future short-term rates are equal to current short-term rates.
- The segmentation hypothesis contends that borrowers and lenders are constrained to particular segments of the yield curve.

BKM 14.3

Under the expectations hypothesis, if the yield curve is upward sloping, the market must expect an increase in short-term interest rates. True / false / uncertain? Why?

BKM 14.4

Under the liquidity preference theory, if inflation is expected to be falling over the next few years, long-term interest rates will be higher than short-term rates. True / false / uncertain? Why?

BKM 14.6

Assuming that the expectations hypothesis is valid, compute the expected price path of the four-year bond in Problem 5 as time passes. What is the rate of return of the bond in each year? Show that the expected return equals the forward rate for each year.

BKM 14.7

The following table shows yields to maturity of U.S. Treasury securities as of January 1, 1993 :

Term to Maturity (in years)	Yield to Maturity
1	3.50%
2	4.50
3	5.00
4	5.50
5	6.00
10	6.50

- Based on the data in the table, calculate the implied forward one-year rate of interest at January 1, 1996.
- Describe the conditions under which the calculated forward rate would be an unbiased estimate of the one-year spot rate of interest at January 1, 1996.

Assume that one year earlier, at January 1, 1992, the prevailing term structure for U.S. Treasury securities was such that the implied forward one-year rate of interest at January 1, 1996, was significantly higher than the corresponding rate implied by the term structure at January 1, 1993.

c. On the basis of the pure expectations theory of the term structure, briefly discuss *two* factors that could account for such a decline in the implied forward rate.

BKM 14.9

The current yield curve for default free zero-coupon bonds is as follows:

Maturity (Years)	YTM
1	10%
2	11
3	12

- a. What are the implied forward one-year rates?
- b. Assume that the pure expectations hypothesis of the term structure is correct. If market expectations are accurate, what will the pure yield curve, that is, the yields to maturity on one- and two-year zero-coupon bonds, be next year?
- c. If you purchased a two-year zero-coupon bond now, what is the expected total rate of return over the next year? If it were a three-year zero-coupon bond? (Hint: Compute the current and expected future prices.) Ignore taxes.
- d. What should be the current price of a three-year maturity bond with a 12% coupon rate paid annually? If you purchased it at that price, what would your total expected rate of return be over the next year (coupon plus price change)? Ignore taxes.

BKM 14.14

You observe the following term structure:

	<u>Effective Annual YTM</u>
1-year zero-coupon bond	6.1%
2-year zero-coupon bond	6.2
3-year zero-coupon bond	6.3
4-year zero-coupon bond	6.4

- a. If you believe that the term structure next year will be the same as today's, will the one-year or the four-year zeros provide a greater expected one-year return?
- b. What if you believe in the expectations hypothesis?

Solutions:

BKM 14.1

Expectations Hypothesis

The yields on long-term bonds are geometric averages of present and expected future short rates. An upward sloping curve is explained by expected future short rates being higher than the current short rate. A downward-sloping yield curve implies expected future short rates are lower than the current short rate. Thus bonds of different maturities have different yields if expectations of future short rates are different from the current short rate.

Liquidity Preference Hypothesis

Yields on long-term bonds are greater than the expected return from rolling over short-term bonds in order to compensate investors in long-term bonds for bearing interest rate risk. Thus bonds of different maturities can have different yields even if expected future short rates are all equal to the current short rate. An upward sloping yield can be consistent even with expectations of falling short rates if liquidity premiums are high enough. If, however, the yield curve is downward sloping and liquidity premiums are assumed to be positive, then we can conclude that future short rates are expected to be lower than the current short rate.

Segmentation Hypothesis

This hypothesis would explain a sloping yield curve by an imbalance between supply and demand for bonds of different maturities. An upward sloping yield curve is evidence of supply pressure in the long-term market and demand pressure in the short-term market. According to the segmentation hypothesis, expectations of future rates have little to do with the shape of the yield curve.

BKM 14.2

b. The long-term rate is an average of the anticipated short-term rates *and* the current short-term rate.

BKM 14.3

True. Under the expectations hypothesis, there are no risk premia built into bond prices. The only reason for long-term yields to exceed short-term yields is an expectation of higher short-term rates in the future.

BKM 14.4

Uncertain. Lower inflation will usually lead to lower nominal interest rates. Nevertheless, if the liquidity premium is sufficiently great, long-term yields may exceed short-term yields *despite* expectations of falling short rates.

BKM 14.6

Expected price path of 4-year zero coupon bond is as follows. (we discount face value by the appropriate sequence of forward rates implied by this year's yield curve.)

Beginning of Year	Expected Price	Expected Rate of Return
1	\$792.16	6%(839.69/792.16 - 1)
2	$\frac{1000}{(1.05)(1.06)(1.07)} = 839.69$	5%(881.68/839.69 - 1)
3	$\frac{1000}{(1.06)(1.07)} = 881.68$	6%(934.58/881.68 - 1)
4	$\frac{1000}{1.07} = 934.58$	7%(1000/934.58 - 1)

BKM 14.7

a.

$$\begin{aligned}
 (1 + y_4)^4 &= (1 + y_3)^3 (1 + f_4) \\
 (1.055)^4 &= (1.05)^3 (1 + f_4) \\
 1.2388 &= 1.1576 (1 + f_4) \\
 f_4 &= .0701 \text{ or } 7.01\%
 \end{aligned}$$

b. The conditions would be those that underlie the pure expectations theory of the term structure: risk neutral market participants who are willing to substitute among maturities solely on the basis of yield differentials. This behavior would rule out liquidity or term premia relating to risk as well as market segmentation based on maturity preferences.

c. Under the expectations hypothesis, lower implied forward rates would indicate lower expected future spot rates for the corresponding period. Since the lower expected future rates embodied in the term structure

are nominal rates, either lower expected future real rates or lower expected future inflation rates would be consistent with the specified change in the observed (implied) forward rate.

BKM 14.9

a. We obtain forward rates from the following table:

Maturity	YTM	Price	Forward Rate
1	10%	909.09	
2	11%	811.62	12.01% $(1.11^2/1.10 - 1)$
3	12%	711.78	14.03% $(1.12^3/1.11^2 - 1)$

b. We obtain next year's prices and yields by discounting each zero's face value at the forward rates for next year that we derived in part (a):

BKM 14.14

a. The return on the 1 year bond will be 6.1%. The price of the 4 year zero today is $\$1000/1.064^4 = \780.25 . Next year, if the yield curve is unchanged, the bond will have a 3 year maturity, a YTM of 6.3%, and therefore sell for $\$1000/1.063^3 = \832.53 , resulting in a one year return of 6.7%. The longer term bond gave the higher return in this case because its YTM fell during the holding period.

b. If you believed in the expectations theory, you could not believe that the yield curve next year will be the same as today's curve. the upward slope in today's curve would be evidence that expected short rates are rising and that the yield curve will shift upward, reducing the holding period return on the 4 year bond. Under the expectations hypothesis, all bonds have equal expected holding period returns. Therefore, you would predict that the HPR for the 4 year bond would be 6.1%, the same as for the 1 year bond.