Practice Problems With Solutions For Topic 12

Problems: Bodie Kane Marcus: 8.1 to 8.17

BKM 8.1

What is the beta of a portfolio with $E(r_p) = 20\%$, if $r_f = 5\%$ and $E(r_M) = 15\%$?

BKM 8.2

The market price of a security is $\$40$. Its expected rate of return is 13%. The risk-free rate is 7% and the market risk premium is 8%. What will be the market price of the security if its covariance with the market portfolio doubles (and all other variables remain unchanged)? Assume that the stock is expected to pay a constant dividend in perpetuity.

BKM 8.3

You are a consultant to a large manufacturing corporation that is considering a project with the following net after-tax cash flows (in millions of dollars):

Years from Now	After-Tax Cash Flow
0	-20
$1-9$	10
10	20

The project's beta is 1.7. Assuming that $r_f = 9\%$ and $E(r_M) = 19\%$, what is the net present value of the project? What is the highest possible beta estimate for the project before its NPV becomes negative?

BKM 8.4

Are the following true or false?

a. Stocks with a beta of zero offer an expected rate of return of zero.

b. The CAPM implies that investors require a higher return to hold highly volatile securities.

c. You can construct a portfolio with a beta of 0.75 by investing 0.75 of the investment budget in bills and the remainder in the market portfolio.

BKM 8.5

Consider the following table, which gives a security analyst's expected return on two stocks for two particular market returns:

Market Return	Aggressive Stock	Defensive Stock
5%	2%	3.5%
20	32	14

a. What are the betas of the two stocks?

b. What is the expected rate of return on each stock if the market return is equally likely to be 5% or 20%?

c. If the T-bill rate is 8% and the market return is equally likely to be 5% or 20%, draw the SML for this economy.

d. Plot the two securities on the SML graph. What are the alphas of each?

e. What hurdle rate should be used by the management of the aggressive firm for a project with the risk characteristics of the defensive firm's stock?

If the simple CAPM is valid, which of the following situations in Problems 6 to 12 are possible? Explain. Consider each situation independently.

BKM 8.6

Portfolio	Expected Return	Beta
A	20	1.4
B	25	1.2

BKM 8.7

Portfolio	Expected Return	Standard Deviation
A	30	35
B	40	25

BKM 8.8

Portfolio	Expected Return	Standard Deviation
Risk-free	10	0
Market	18	24
A	16	12

BKM 8.9

Portfolio	Expected Return	Standard Deviation
Risk-free	10	0
Market	18	24
A	20	22

BKM 8.10

Portfolio	Expected Return	Beta
Risk-free	10	0
Market	18	1.0
A	16	1.5

BKM 8.11

Portfolio	Expected Return	Beta
Risk-free	10	0
Market	18	1.0
A	16	.9

BKM 8.12

Portfolio	Expected Return	Standard Deviation
Risk-free	10	0
Market	18	24
A	16	22

In Problems 13 to 15 assume that the risk-free rate of interest is 8% and the expected rate of return on the market is 18%.

BKM 8.13

A share of stock sells for \$100 today. It will pay a dividend of \$9 per share at the end of the year. Its beta is 1. What do investors expect the stock to sell for at the end of the year?

BKM 8.14

I am buying a firm with an expected cash flow of \$1,000 but am unsure of its risk. If I think the beta of the firm is zero, when in fact the beta is really 1, how much *more* will I offer for the firm than it is truly worth?

BKM 8.15

A stock has an expected rate of return of 6%. What is its beta? Why would anyone consider bying this risky asset which provides an expected return less than the risk-free rate?

BKM 8.16

Two investment advisors are comparing performance. One averaged a 19% rate of return and the other a 16% rate of return. However, the beta of the first investor was 1.5, whereas that of the second was 1.

a. Can you tell which investor was a better predictor of individual stocks (aside from the issue of general movements in the market)?

b. If the T-bill rate were 6% and the market return during the period were 14%, which investor would be the superior stock selector?

c. What if the T-bill rate were 3% and the market return were 15%?

BKM 8.17

In 1994 the rate of return on short-term government securities (perceived to be risk-free) was about 4%. Suppose the expected rate of return required by the market for a portfolio with a beta measure of 1 is 12%. According to the capital asset pricing model (security market line):

a. What is the expected rate of return on the market portfolio?

b. What would be the expected rate of return on a stock with $\beta = 0$?

c. Suppose you consider buying a share of stock at \$40. The stock is expected to pay \$3 dividends next year and you expect it to sell then for \$41. The stock risk has been evaluated by $\beta = -.5$. Is the stock overpriced or underpriced?

Solutions:

BKM 8.1

$$\begin{aligned} E\ (r_p) &= r_f + \beta [E (r_m) - r_f] \\ 20 &= 5 + \beta (15 - 5) \\ \beta &= 15/10 = 1.5 \end{aligned}$$

BKM 8.2

If the covariance of the security doubles then so will its beta and its risk premium. The current risk premium is 6% $(= 13 - 7)$, so the new risk premium would be 12%, and the new discount rate for the security would be $12 + 7 = 19\%$.

If the stock pays a level perpetual dividend, then we know from the original data that the dividend, D, must satisfy the equation for a perpetuity:

$$\begin{aligned} \text{Price} &= \text{Dividend / Discount rate} \\ 40 &= D/.13 \\ D &= 40(.13) = \$5.20 \end{aligned}$$

At the new discount rate of 19%, the stock would be worth only $\$5.20 / .19 = \$27.37$. The increase in stock risk has lowered its value by 31.58%.

BKM 8.3

The appropriate discount rate for the project is:

$$r_f + \beta \left[E\left(r_m\right) - r_f\right] = 9 + 1.7(19 - 9) = 26\%$$

Using this discount rate,

$$\begin{aligned} NPV &= -20 + \sum_{t=1}^{10} \frac{10}{1.26^t} + \frac{10}{1.26^{10}} \\ &= -20 + 10 \times PA(26\%,\ 10 \text{ years }) + \\ &\quad 10 \times PF(26\%,\ 10 \text{ years }) \\ &= 15.64 \end{aligned}$$

The internal rate of return on the project is 49.55%. The highest value that beta can take before the hurdle rate exceeds the IRR is determined by

$$\begin{aligned} 49.55 &= 9 + \beta(19 - 9) \\ \beta &= 40.55/10 = 4.055 \end{aligned}$$

BKM 8.4

a. False. $\beta = 0$ implies $E(r) = r_f$, not zero.

b. False. Investors require a risk premium only for bearing systematic risk (undiversifiable or market) risk.

c. False. 75% of your portfolio should be in the market, and 25% in bills. Then,

$$\beta_p = .75 \times 1 + .25 \times 0 = .75$$

BKM 8.5

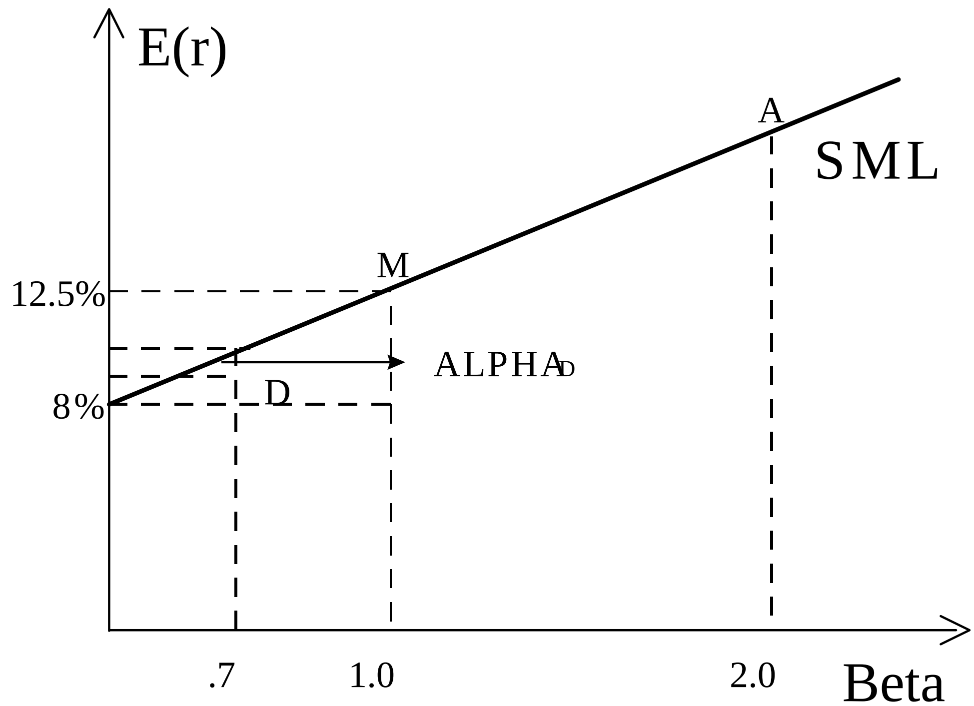
a. The beta is the sensitivity of the stock's return to the market return. Call A the aggressive stock and D the defensive one. Then beta is the change in the stock return per change in the market return. Therefore, we compute each stock's beta by calculating the difference in its return across the two scenarios divided by the difference in the market return.

$$\beta_A = \frac{2 - 32}{5 - 20} = 2.00$$
$$\beta_D = \frac{3.5 - 14}{5 - 20} = .70$$

b. With equal likelihood of either scenarios, the expected return is an average of the two possible outcomes.

$$E(r_A) = .5(2 + 32) = 17\%$$
$$E(r_B) = .5(3.5 + 14) = 8.75\%$$

c. The SML is determined by the market expected return of $.5(20 + 5) = 12.5\%$, with a beta of 1, and the bill return of 8% with a beta of zero. See the following graph.



The equation for the security market line is:

$$E(r) = 8 + \beta(12.5 - 8)$$

d. The aggressive stock has a fair expected return of: $E(r_A) = 8 + 2.0(12.5 - 8) = 17\%$ and the expected return by the analyst is also 17%. Thus its alpha is zero. Similarly, the required return on the defensive stock is: $E(r_D) = 8 + .7(12.5 - 8) = 11.15\%$, but the analyst's expected return on D is only 8.75%, and hence

$$\alpha_D = \text{ actual expected return - required return (given risk)}$$
$$= 8.75 - 11.15 = -2.4\%$$

The points for each stock plot on the graph as indicated above.

e. The hurdle rate is determined by the project beta, $.7$, not by the firm's beta. The correct discount rate is 11.15%, the fair rate of return on stock D.

BKM 8.6

Not possible. Portfolio A has a higher beta than B, but its expected return is lower. Thus, A cannot exist in equilibrium.

BKM 8.7

Possible. If the CAPM is valid, the expected rate of return compensates only for systematic (market) risk represented by beta rather than the standard deviation which includes nonsystematic risk. Thus, A's lower rate of return can be paired with a higher standard deviation, as long as A's beta is lower than B's.

BKM 8.8

Not possible. The reward-to-variability ratio for portfolio A is better than that of the market, which is impossible according to CAPM, since the CAPM predicts that the market is the most efficient portfolio. Using the numbers supplied,

$$S_A = \frac{16-10}{12} = .5 \qquad S_M = \frac{18-10}{24} = .33$$

The numbers would imply that portfolio A provides a better risk-reward trade-off than the market portfolio.

BKM 8.9

Not possible. Portfolio A clearly dominates the market portfolio. It has a lower standard deviation with a higher expected return.

BKM 8.10

Not possible. The SML for this situation is: $E(r) = 10 + \beta(18-10)$

Portfolios with beta of 1.5 have an expected return of $E(r) = 10 + 1.5(18-10) = 22\%$.

A's expected return is 16%, that is, A plots below the SML (has a negative alpha of -6%), and hence, is an overpriced portfolio. This is inconsistent with the CAPM.

BKM 8.11

Not possible. Same SML as in problem 10. Here portfolio A's required expected return is: $10+.9(8) = 17.2\%$, which is still higher than 16%. A is overpriced with a negative alpha of -1.2%.

BKM 8.12

Possible. Same CML as shown in problem 8. Portfolio A plots below the CML, as any asset is expected to. This situation is not inconsistent with the CAPM.

BKM 8.13

Since the stock's beta is equal to 1, its expected rate of return should be equal to that of the market, that is, 18%.

$$E(r) = \frac{D + P_1 - P_0}{P_0}$$
$$18 = \frac{9 + P_1 - 100}{100}$$
$$P_1 = \$109$$

BKM 8.14

Assume that the $\$1,000$ is a perpetuity. If beta is zero, the cash flow should be discounted at the risk-free rate, 8%.

$$PV = 1000\,/.08 = \$12,500$$

If, however, beta is equal to 1, the investment should yield 18%, and the price paid for the firm should be:

$$PV = 1000\,/.18 = \$5,555.56$$

The difference, $\$6,944.44$, is the amount you will overpay if you erroneously assumed that beta is zero rather than 1.

If the cash flow lasts only one year:

$$PV(\beta = 0) = 1000/(1+.08) = \$925.93$$
$$PV(\beta = 1) = 1000/(1+.18) = \$847.46$$

with a difference of $\$78.47$.

For any n year cash flow the difference is:

$$1,000 \frac{P}{\frac{1}{8\%}\left[1 - \frac{1}{1.08^n}\right]} - 1,000 \frac{P}{\frac{1}{8\%}\left[1 - \frac{1}{1.18^n}\right]}$$

BKM 8.15

Using the SML: $6 = 8 + \beta(18 - 8)$, we get that

$$\beta = -2/10 = -.2$$

This asset has a negative market beta, i.e. its return is negatively correlated with that of the market. This asset can thus serve as an insurance against systematic risk. Instead, the return on the risk-free asset is simply uncorrelated with the market return.This is why the stock has a lower expected return.

BKM 8.16

$r_1 = 19\%$; $r_2 = 16\%$; $\beta_1 = 1.5$; $\beta_2 = 1$

a. To tell which investor was a better predictor of individual stocks we look at their abnormal return, which is the ex-post alpha, that is, the abnormal return is the difference between the actual return and that predicted by the SML. Without information about the parameters of this equation (risk-free rate and market rate of return) we cannot tell which one is more accurate.

b. If $r_f = 6\%$ and $r_M = 14\%$, then (using the notation of alpha for the abnormal return)

$$\alpha_1 = 19 - [6 + 1.5\,(14 - 6)] = 19 - 18 = 1\%$$
$$\alpha_2 = 16 - [6 + 1\,(14 - 6)] = 16 - 14 = 2\%$$

Here, the second investor has the larger abnormal return and thus he appears to be a more accurate predictor. By making better predictions the second investor appears to have tilted his portfolio toward underpriced stocks.

c. If $r_f = 3\%$ and $r_M = 15\%$, then

$$\alpha_1 = 19 - [3 + 1.5\,(15 - 3)] = 19 - 21 = -2\%$$
$$\alpha_2 = 16 - [3 + 1\,(15 - 3)] = 16 - 15 = 1\%$$

Here, not only does the second investor appear to be a better predictor, but the first investor's predictions appear valueless (or worse).

BKM 8.17

a. Since the market portfolio by definition has a beta of 1, its expected rate of return is 12%.

b. $\beta = 0$ means no systematic risk. Hence, the portfolio's fair return is the risk-free rate, 4%.

c. Using the SML, the *fair* rate of return of a stock with $\beta = -0.5$ is:

$$E(r) = 4 + (-.5)(12 - 4) = 0\%$$

The *expected* rate of return, using the expected price and dividend of next year:

$$E(r) = 44/40 - 1 = .10, \text{ or } 10\%$$

Because the expected return exceeds the fair return, the stock must be underpriced.