M.I.T. **Fall 1998**

Sloan School of Management **15.415**

Prof. Denis Gromb

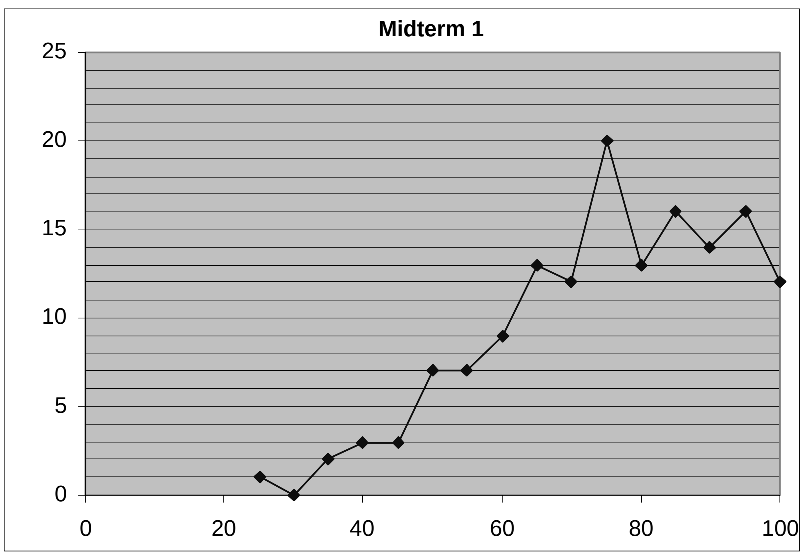
SOLUTION TO MIDTERM 1

Section B

Number of students: 148

Mean: 72

Grade Distribution:



Letter grades: The class is **not** graded on a curve. The following are approximations:

- A: from 75 to 100
- B: from 60 to 75
- C: from 40 to 60
- D: from 20 to 40
- E: below 20

Question 1

1) b; **2)** e; **3)** d; **4)** a

Question 2

a) The three projects' NPV are

$$\begin{aligned}
\text{NPV}_{\text{Atlanta}} &= -135 + \frac{50}{1.07} + \frac{50}{1.08^2} + \frac{50}{1.07^3} = -\$4.59M \\
\text{NPV}_{\text{Boston}} &= -100 + \frac{45}{1.07} + \frac{45}{1.08^2} + \frac{45}{1.07^3} = \$17.37M \\
\text{NPV}_{\text{Chicago}} &= -190 + \frac{50}{1.07} + \frac{100}{1.08^2} + \frac{100}{1.07^3} = \$24.09M
\end{aligned}$$

These being independent projects, we should undertake all projects with a positive NPV. Hence, we should open new branches in Boston and Chicago.

b) Financial markets being frictionless, investment decisions are independent of wealth.. Hence, the answer as the same as in a): LHS Inc. should open a new branch in Boston and Chicago. (This might for instance involve borrowing against the project's future cash flows).

c) Among mutually exclusive projects, we should undertake the one with the highest NPV if it is positive. Hence, we should open a new branch in Chicago.

d) Using the IRR function of the calculator, we find

$$\text{IRR}_{\text{Atlanta}} = 5.46\% \qquad \text{IRR}_{\text{Boston}} = 16.65\% \qquad \text{IRR}_{\text{Chicago}} = 13.52\%$$

The IRR rule with a 14% threshold would lead to open a branch in Boston, but not Chicago.

Note: We do not need to compute the IRR of each project. Indeed, if we compute the NPV of each project using 14% as the discount rate, we find:

$$\begin{aligned}
\text{NPV}_{\text{Atlanta}}(14\%) &= -135 + \frac{50}{1.14} + \frac{50}{1.14^2} + \frac{50}{1.14^3} = -\$18.92M < 0 \\
\text{NPV}_{\text{Boston}}(14\%) &= -100 + \frac{45}{1.14} + \frac{45}{1.14^2} + \frac{45}{1.14^3} = \$4.47M > 0 \\
\text{NPV}_{\text{Chicago}}(14\%) &= -190 + \frac{50}{1.14} + \frac{100}{1.14^2} + \frac{100}{1.14^3} = -\$1.70M < 0
\end{aligned}$$

Hence, $\text{IRR}_{\text{Atlanta}} < 14\%$, $\text{IRR}_{\text{Boston}} > 14\%$ and $\text{IRR}_{\text{Chicago}} < 14\%$ and the IRR rule with a 14% threshold would lead to open a branch in Boston, but not Chicago.

Question 3

a) We must equate the PV of the payments and that of the withdrawals.

$$\text{PV}_{\text{Payments}} = X + \frac{X}{5\%}\left(1 - \frac{1}{(1.05)^{19}}\right) = X \cdot \left[1 + \frac{1}{5\%}\left(1 - \frac{1}{(1.05)^{19}}\right)\right]$$

$$\text{PV}_{\text{Withdrawals}} = \frac{1}{(1.05)^{24}} \cdot \frac{1,000}{5\%}\left(1 - \frac{1}{(1.05)^5}\right)$$

Hence

$$X = \frac{\frac{1}{(1.05)^{24}} \cdot \frac{1,000}{5\%}\left(1 - \frac{1}{(1.05)^5}\right)}{1 + \frac{1}{5\%}\left(1 - \frac{1}{(1.05)^{19}}\right)} = \$102.6$$

b) We must equate the PV of the payments and that of the withdrawals.

$$\text{PV}_{\text{Withdrawals}} = \frac{1}{(1.05)^{24}} \cdot \frac{1,000}{5\% - 4\%}\left(1 - \left(\frac{1.04}{1.05}\right)^5\right)$$

Hence

$$X = \frac{\frac{1}{(1.05)^{24}} \cdot \frac{1,000}{5\%-4\%}\left(1 - \left(\frac{1.04}{1.05}\right)^5\right)}{1 + \frac{1}{5\%}\left(1 - \frac{1}{(1.05)^{19}}\right)} = \$110.7$$

c) The real rate ρ obtains from the nominal rate $r = 5\%$ and the inflation rate $i = 2\%$ by

$$\rho = \frac{1+r}{1+i} - 1 = 2.94\%$$

d) We must equate the PV of the payments into the fund and that of the granddaughter's withdrawals. We should be careful to discount real cash flows using the real discount rate and nominal cash flows using nominal discount rates.

$$\text{PV}_{\text{Payments}} = Y \cdot \left[1 + \frac{1}{\rho}\left(1 - \frac{1}{(1+\rho)^{19}}\right)\right]$$

Hence

$$Y = \frac{\frac{1}{(1.05)^{24}} \cdot \frac{1,000}{5\%}\left(1 - \frac{1}{(1.05)^5}\right)}{1 + \frac{1}{2.94\%}\left(1 - \frac{1}{(1.0294)^{19}}\right)} = \$87.2$$

Question 4

a) We need to form a portfolio that has the same PV and duration as the liability.

$$\text{PV}_{\text{Liability}} = \frac{2}{7\%} \cdot \left[1 - \frac{1}{(1.07)^{25}}\right] = \$23.3M$$

$$\text{D}_{\text{Liability}} = \frac{1.07}{.07} - \frac{25}{(1.07)^{25} - 1} = 9.64 \text{ years}$$

We know that the price and duration of a zero with face value P and maturity T are:

$$B = \frac{P}{1.07^T} \text{ and } D = T$$

Hence, you need to invest $\$23.3M$ into a zero coupon bond with maturity 9.64 years and face value $P = 23.3 \times (1.07^{9.64}) = \$44.73M$.

b) Again, we need to form a portfolio that has the same PV and duration as the liability. The duration of a portfolio of 5-year and 20-year zeros is

$$D = 20w + 5(1-w) = 15w + 5$$

where w is the fraction of the portfolio's value that is invested in the 20-year bond. This weight is thus the solution to

$$15w + 5 = 9.64$$

so that $w = 0.31$. Hence, the portfolio is formed by investing

$$\begin{cases} w \times 23.3M = \$7.22M \text{ in the 20-year zero} \\ (1-w) \times 23.3M = \$16.08M \text{ in the 5-year zero} \end{cases}$$

Question 5

a) Let B_1 and B_2 denote the prices of the 1-year and 2-year zeros with face value $\$1$. The absence of arbitrage opportunities implies that they are solutions to:

$$\begin{cases} B_A = 50B_1 + 1050B_2 \\ B_B = 100B_1 + 1100B_2 \end{cases} \text{ or } \begin{cases} 2B_A - B_B = 1000B_2 \\ 100B_1 = B_B - 1100B_2 \end{cases}$$

$$\begin{cases} B_2 = \frac{1}{1000}\left[2B_A - B_B\right] = 0.8573 \\ B_1 = \frac{1}{100}\left[B_B - 1100B_2\right] = 0.9347 \end{cases} \text{ so that } \begin{cases} r_1 = \frac{1}{B_1} - 1 = 7\% \\ r_2 = \frac{1}{(B_2)^{\frac{1}{2}}} - 1 = 8\% \end{cases}$$

Hence, the forward rate between year 1 and 2 is

$$f_2 = \frac{B_1}{B_2} - 1 = \frac{(1+r_2)^2}{1+r_1} - 1 = 9\%$$

b) We first need to determine the number of bonds that $\$12,000$ can buy. Hence, we need to determine the price of bond C

$$B_C = \frac{30}{1.07} + \frac{1030}{1.08^2} = \$911.1$$

Hence, $\$12,000$ can buy

$$\frac{12,000}{B_C} = 13.17$$

Hence, the cash flows that this investment generate one year and two years from today are

$$\begin{aligned} C_1 &= 13.17 \times 30 = \$395.1 \\ C_2 &= 13.17 \times 1030 = \$13,565.1 \end{aligned}$$