M.I.T.
Sloan School of Management

Prof. Denis Gromb

Fall 1998
15.415

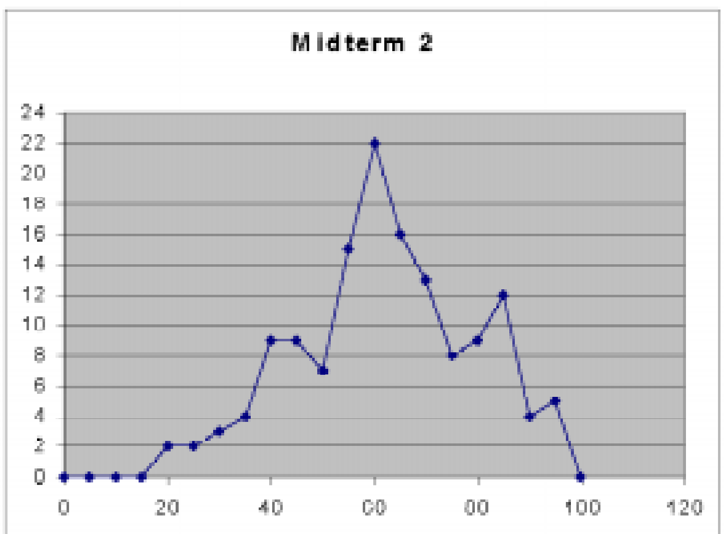
MIDTERM 2

Section A

Number of students: 140

Mean: 65

Grade Distribution:



Letter grades: The class is **not** graded on a curve. The following are approximations:

- A: from 70 to 100
- B: from 55 to 70
- C: from 35 to 55
- D: from 20 to 35
- E: below 20

Question 1

1) e **2)** a and c **3)** b and c **4)** a

Question 2

a) The return is $\frac{25-50}{50} = -50\%$ with probability 0.3 and $\frac{75-50}{50} = 50\%$ with probability 0.7.

Expected return and variance are given by:

$$\begin{aligned} \mathsf{E}[r_A] &= 0.3 \times (-50\%) + 0.7 \times 50\% = 20\% \\ \sigma_A^2 &= \mathsf{E}\left[(r_A - \mathsf{E}[r_A])^2\right] = 0.3 \times (-50\% - 20\%)^2 + 0.7 \times (50\% - 20\%)^2 = 0.21 \end{aligned}$$

b) The return is $\frac{15-5.45}{5.45} = 175.23\%$ with probability 0.3 and $\frac{0-5.45}{5.45} = -100\%$ with probability 0.7.

Expected return and variance are given by:

$$\begin{aligned} \mathsf{E}[r_B] &= 0.3 \times 175.23\% + 0.7 \times (-100\%) = -17.43\% \\ \sigma_B^2 &= \mathsf{E}\left[(r_B - \mathsf{E}[r_B])^2\right] = 0.3 \times (175.23\% + 17.43\%)^2 + 0.7 \times (-100\% + 17.43\%)^2 = 1.59 \end{aligned}$$

c) The correlation should be negative because we know that as the stock price increases, the price of the put decreases.

The correlation coefficient between the stock return and the put option's return is given by:

$$\begin{aligned} \rho_{AB} &= \frac{\mathsf{E}[(r_A - \mathsf{E}[r_A])(r_B - \mathsf{E}[r_B])]}{\sigma_A \sigma_B} \\ &= \frac{0.3 \times (-50\% - 20\%)(175.23\% + 17.43\%) + 0.7 \times (50\% - 20\%)(-100\% + 17.43\%)}{\sqrt{0.21}\sqrt{1.59}} \\ &= -1 \end{aligned}$$

d) The portfolio weight on the shares is

$$w = \frac{2 \cdot 50}{2 \cdot 50 + 10 \cdot 5.45} = 0.647$$

Hence the portfolio's expeceted return and variance are

$$\begin{aligned} \mathsf{E}[r_p] &= w\mathsf{E}[r_A] + (1-w)\mathsf{E}[r_B] = 6.69\% \\ \sigma_p^2 &= w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\sigma_{AB} \\ &= w^2\sigma_A^2 + (1-w)^2\sigma_B^2 + 2w(1-w)\sigma_A\sigma_B\rho_{AB} = 0.022 \end{aligned}$$

Question 3

a) XYZ's stock price is the sum of two terms:

- a 6-year annuity with a first payment of $(1.02 \cdot 5)$ one year from today and growing at rate 2%
- a perpetuity with flat payment of $(1.02^6 \cdot 5)$ with first payment seven years from today

$$P_{XYZ} = \frac{1.02 \cdot 5}{7\% - 2\%} \cdot \left[1 - \frac{1.02^6}{1.07^6}\right] + \frac{1}{1.07^6} \cdot \frac{1.02^6 \cdot 5}{7\%} = \$79.06$$

ABC's price is given by $P_{ABC} = \frac{(1+g)\cdot 7}{9\% - g} = 100$

Hence, $g = \frac{9\% \cdot 100 - 7}{100+7} = 1.87\%$

b) HAL's market capitalization is:

$$5\% \times [500,000 \times P_{XYZ}] + 10\% \times [200,000 \times P_{XYZ}] = \$3,976,500$$

Hence its share price is

$$P_{HAL} = \frac{3,976,500}{100,000} = \$39.76$$

Its PVGO is zero since its plowback ratio is zero.

Question 4

a) We note that $R = 1.02$, $u = 1.1$ and $d = 0.8$ so that $q = \frac{R-d}{u-d} = 0.733$

The European put's payoffs at maturity are: $P_{uu} = 0$, $P_{ud} = \$1$ and $P_{dd} = \$13$. Hence:

$$P = \frac{1}{R^2}\left[q^2 P_{uu} + 2q(1-q)P_{ud} + (1-q)^2 P_{dd}\right] = \$1.26$$

To find the American put's premium, we need to determine whether it might be exercised prior to maturity. If the stock price goes up to $S_u = \$55$, the put is out of the money (since $45 < 55$), and so it is not optimal to exercise it. Hence $p_u = \frac{1}{R}[qP_{uu} + (1-q)P_{ud}] = \$0.26$. If the stock price goes down to $S_d = \$40$, exercising the put would yield a $\$5$ payoff. Instead, if the put were held to maturity, it would be worth

$$\frac{1}{R}[qP_{ud} + (1-q)P_{dd}] = \$4.12$$

Hence, it is optimal to exercise the put and so $p_d = \$5$. Finally, the put is currently out of the money (since $45 < 50$), and so it is not optimal to exercise it immediately (at $t = 0$). Hence

$$p = \frac{1}{R}[qp_u + (1-q)p_d] = \$1.50$$

b) Since there are no dividends, both calls have the same price and by put-call parity:

$$c = C = S + P - \frac{45}{R^2} = \$8.01$$

Question 5

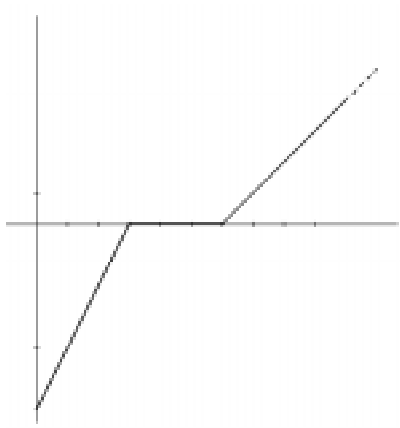
a) One such portfolio (among many others) is:

- Short a risk-free bond with $\$30$ face value and 2-month maturity
- 1 long European call on one XYZ share with 2-month maturity and strike price $\$40$
- 2 written European calls on one XYZ share with 2-month maturity and strike price $\$50$
- 1 long European call on one XYZ share with 2-month maturity and strike price $\$60$

b) One such portfolio (among many others) is:

- 1 long European put on one XYZ share with 2-month maturity and strike price $\$50$
- 1 long European call on one XYZ share with 2-month maturity and strike price $\$20$
- 1 short European call on one XYZ share with 2-month maturity and strike price $\$50$
- 1 long European call on one XYZ share with 2-month maturity and strike price $\$70$

c)



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