M.I.T. **Fall 1998**
Sloan School of Management **15.415**

Prof. Denis Gromb

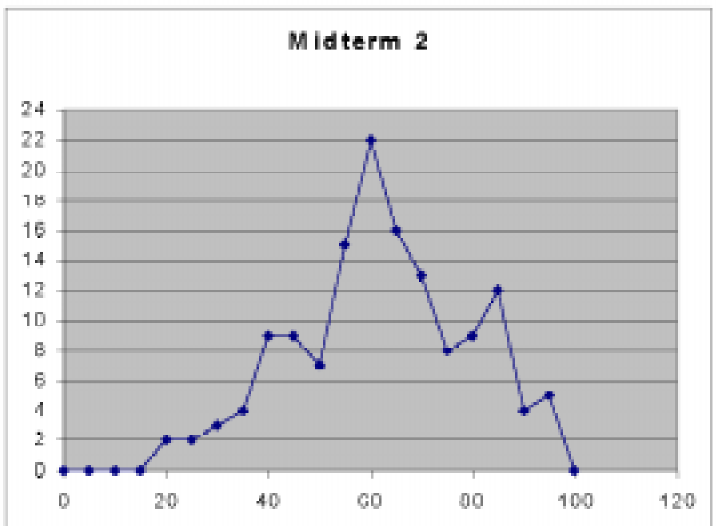
MIDTERM 2

Section B

Number of students: 140

Mean: 65

Grade Distribution:



Letter grades: The class is **not** graded on a curve. The following are approximations:

- A: from 70 to 100
- B: from 55 to 70
- C: from 35 to 55
- D: from 20 to 35
- E: below 20

Question 1

1) b and c **2)** b and c **3)** a **4)** a

Question 2

Expected returns:

$$\begin{aligned}\mathsf{E}[r_A] &= \frac{1}{3}\cdot 10\% + \frac{1}{3}\cdot 8\% + \frac{1}{3}\cdot 6\% = 8\% \\ \mathsf{E}[r_B] &= \frac{1}{3}\cdot 10\% + \frac{1}{3}\cdot 4\% + \frac{1}{3}\cdot 7\% = 7\%\end{aligned}$$

Variances:

$$\begin{aligned}\sigma_A^2 &= \mathsf{E}\left[(r_A - \mathsf{E}[r_A])^2\right] = \frac{1}{3}\cdot(10\% - 8\%)^2 + \frac{1}{3}\cdot(8\% - 8\%)^2 + \frac{1}{3}\cdot(6\% - 8\%)^2 \\ &= 0.0002667 \\ \sigma_B^2 &= \mathsf{E}\left[(r_B - \mathsf{E}[r_B])^2\right] = \frac{1}{3}\cdot(10\% - 7\%)^2 + \frac{1}{3}\cdot(4\% - 7\%)^2 + \frac{1}{3}\cdot(7\% - 7\%)^2 \\ &= 0.0006000\end{aligned}$$

Covariance:

$$\begin{aligned}\sigma_{AB} &= \mathsf{E}\left[(r_A - \mathsf{E}[r_A])(r_B - \mathsf{E}[r_B])\right] \\ &= \frac{1}{3}\cdot(10\% - 8\%)(10\% - 7\%) + \frac{1}{3}\cdot(8\% - 8\%)(4\% - 7\%) + \frac{1}{3}\cdot(6\% - 8\%)(7\% - 7\%) \\ &= 0.0002000\end{aligned}$$

b) The portfolio with expected return 7.5% has equal weights on assets A and B because

$$7.5\% = \frac{1}{2}\cdot 8\% + \frac{1}{2}\cdot 7\%$$

The variance of its return is thus:

$$\sigma^2 = \left(\frac{1}{2}\right)^2 \sigma_A^2 + \left(\frac{1}{2}\right)^2 \sigma_B^2 + 2\cdot\frac{1}{2}\cdot\frac{1}{2}\sigma_{AB} = 0.000317$$

Compared to asset A, this portfolio's return has a lower mean and a higher variance. Hence, no investor will want to invest all their wealth in it.

Question 3

a) The value of one share is

$$P = \frac{EPS_1}{r} = \frac{10}{10\%} = \$100$$

It is also given by

$$P = \frac{D}{r_q}$$

where r_q, the discount rate for a quarter, is given by $(1+r_q)^4 = (1+r)$. Hence,

$$D = P \cdot \left[(1+r)^{\frac{1}{4}} - 1\right] = 100 \cdot \left[1.1^{0.25} - 1\right] = \$2.41$$

XYZ's PVGO is:

$$PVGO = P - \frac{EPS_1}{r} = \$0$$

b) The value of one share is

$$P = \frac{1}{(1+r)^5} \cdot \frac{D}{r_q - g}$$

Hence

$$g = r_q - \frac{1}{(1+r)^5} \cdot \frac{D}{P} = \left[(1+r)^{\frac{1}{4}} - 1\right] - \frac{1}{1.1^5}\frac{7}{100} = -1.9\%$$

Shareholders are indifferent.

Question 4

a) We note that $R = 1.1$, $u = 1.5$ and $d = 0.5$ so that $q = \frac{R-d}{u-d} = 0.6$.

Debt being senior with respect to equity, we have:

$$S_{uu} = \frac{112.5M - 40M}{500,000} = \$145,\ S_{ud} = S_{dd} = 0$$

Hence,

$$\begin{aligned} S_u &= \frac{1}{R}\left[qS_{uu} + (1-q)S_{ud}\right] = \$79.09 \\ S_d &= \frac{1}{R}\left[qS_{ud} + (1-q)S_{dd}\right] = \$0 \\ S &= \frac{1}{R}\left[qS_u + (1-q)S_d\right] = \$43.14 \end{aligned}$$

b) European put's payoffs at maturity: $P_{uu} = 0$, $P_{ud} = P_{dd} = \$50$

Hence, the premium is:

$$P = \frac{1}{R^2}\left[q^2 P_{uu} + 2q(1-q)P_{ud} + (1-q)^2 P_{dd}\right] = \$26.45$$

To find the American put's premium, we need to determine whether it might be exercised prior to maturity. If the stock price goes up to $S_u = \$79.09$, the put is out of the money (since $50 < 79.09$), and so it is not optimal to exercise it. Hence $p_u = \frac{1}{R}\left[qP_{uu} + (1-q)P_{ud}\right] = \$18.18$. If the stock price goes down to $S_d = \$0$, exercising the put will never yield a higher payoff and so it is optimal to exercise it (if only to gain the time value of money). Hence, $p_d = \$50$. Finally, exercising the put at $t = 0$ would yield $50 - 43.14 = \$6.86$. Hence, it is optimal not to exercise the put initially because

$$\frac{1}{R}\left[qp_u + (1-q)p_d\right] = \$28.10 > \$6.86$$

Hence, $p = \$28.10$.

Question 5

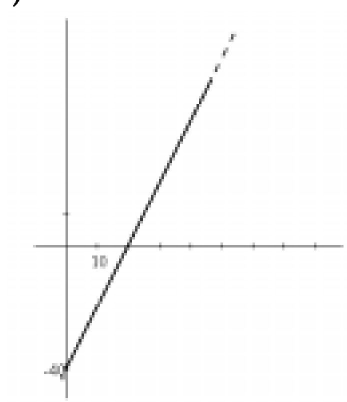
a) One such portfolio (among many others) is:

- 2 long European calls on one share of XYZ with maturity 2 months and strike $\$20$
- 3 written European calls on one share of XYZ with maturity 2 months and strike $\$40$
- 1 long European call on one share of XYZ with maturity 2 months and strike $\$80$

b) One such portfolio (among many others) is:

- 1 long European call on one share of XYZ with maturity 2 months and strike $\$20$
- 2 written European calls on one share of XYZ with maturity 2 months and strike $\$40$
- 2 long European calls on one share of XYZ with maturity 2 months and strike $\$50$
- 2 written European calls on one share of XYZ with maturity 2 months and strike $\$60$
- 1 long European call on one share of XYZ with maturity 2 months and strike $\$90$

c)



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