15.415 Finance Theory

Lecture 2: Present Values

Spring 1999

Preliminary Example

You have invented a new search algorithm for computer databases. One company offers you $7000 for the idea, but first you must develop software that implements the idea. A second company offers $500 but does not require you to develop the software. A programmer would require $6000 payable immediately to program your idea. What should you do?

What Did We Learn?

1. \$1 today $\neq$ \$1 tomorrow

2. The net present value is an important determinant of whether to accept or reject the offer

3. We take the deal with the highest NPV

4. The appropriate discount rate is the opportunity cost of capital.

5. The choice may depend on this discount rate.

In-Class Exercise

In reference to the software example above, suppose that you have found a programmer who is willing to accept \$1500 today and \$4500 at the completion of the programming assignment. Recalculate the NPVs of the bids assuming the 20% interest rate for the credit card and the 5% rate on the savings account.

Dealing with Uncertainty

A Fundamental Tenet of Finance: Assets that have the same risk must have the same expected return.

Example (Part A): A friend of yours operates hot dog stands near Fenway Park. Your friend is willing to sell you the rights for operating the stands during the 1997 World Series. The rights cost \$7500 each. If the Red Sox make it to the World Series, you will make \$100,000 per stand. If they don't make it, you make nothing. There is a 10% chance that the Red Sox will make it all the way to the Series. Suppose that you have \$150,000 you can invest in the stands. What is your expected return?

Example (Part B): You happen to know that you can acquire the rights to sell World Series t-shirts at Fenway Park if the Red Sox play. If the Red Sox win, you will receive \$2,000,000; if not, you would get nothing. What is the most you would be willing to pay for this deal? Why?

What Did We Learn?

1. Uncertainty is an inherent part of many financial decisions.

2. Dealing with uncertain projects requires fundamentally different tools than dealing with projects with certain cash flows.

   In particular, decision makers may be risk averse.

3. One way to value a project that has uncertain cash flows is to compare its risk to that of another project.

   If the risks are the same, the returns must be the same, for otherwise you would always take the better deal.

   This is an surprisingly strong observation that will be important later in the semester.

   For now, we must learn the building blocks...

Compounding: Interest on Interest

Compound Interest: When money is invested at compounded interest, interest payments are reinvested to earn more interest in subsequent periods.

By contrast, simple interest pays interest only on the original balance.

Example: Suppose you put \$1000 into a 3-year certificate of deposit at your bank. Calculate the amount of money you have at the end of the 3 years if the interest rate is (a) 5.2% simple interest and (b) 5.00% compounded annually.

In-Class Exercise

A brand-new baby in your family is enamored with the bright penny in your hand. Caught up in the moment, you promise the baby a 300% compounded annual return on the penny. (The baby will have 4 pennies on its first birthday, 16 pennies on its second birthday, etc.) How many pennies do you owe on the baby's 18th birthday? How many pennies would you owe if you had promised only simple interest?

Terminology

Some terminology will be used repeatedly throughout the course. Let's consider a simple example to highlight these terms.

Example: Suppose that we have the following cash flows.

time	0	1	2
cash flow	-100	200	300
discount rate	N/A	5%	6%

Define and give examples of the following:

- discount rate / opportunity cost of capital
- discount factor
- present value
- net present value
- future value

Some General Formulae

Let r_n be the n-year discount rate.

Present Value for yearly cash flows

$$PV = \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)^2} + \ldots + \frac{C_n}{(1+r_n)^n}$$

Net Present Value for yearly cash flows

$$NPV = C_0 + \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)^2} + \ldots + \frac{C_n}{(1+r_n)^n}$$

n-year Future value for yearly cash flows (assume that $r_n = r$ for all n)

$$FV = C_0(1+r)^n + C_1(1+r)^{n-1} + \ldots + C_n$$

Important: These formulas are not a substitute for good intuition!

Examples of Discount Rates

For those considering riskless investments, the riskless rate of interest may represent the appropriate discount rate. (We will consider an example later.)

- Treasury Bills and Bonds – good source, but we need to wait until we get to the fixed income lectures before we can use these...

- Treasury Strips – We can get discount rates directly from these.

  Example: From Aug 28, 1997 (see WSJ insert)

Year	Quoted Rate	Discount Rate
August 98	5.72%	5.80%
August 99	5.94%	6.03%
August 00	6.06%	6.15%
August 01	6.16%	6.25%

  We will learn the difference between the quoted rate and the discount rate in just a moment.

WSJ INSERT ABOUT HERE

In-Class Exercise

Suppose it is August 28, 1997, and you expect to receive the following cash flows. You have determined that the riskless rate is the appropriate discount rate. (For convenience, the discount rates from earlier are repeated.)

Year	Cash Flow	Discount Rate
August 98	\$100M	5.80%
August 99	\$200M	6.03%
August 00	\$150M	6.15%
August 01	\$220M	6.25%

Calculate the PV of these cash flows using the above formulas. Assume that August '98 is one year from today, ...

Monthly Rates

You rent an apartment in Cambridge for 12 months. The rent is $900 per month. Compute the present value of the rent payments, assuming a discount rate of 8%.

APRs and EARs

Example: Suppose that your bank offers a 1-year CD with a 5% interest rate. How much do you earn on each dollar in a year with

- annual compounding?
- semiannual compounding?
- quarterly compounding?
- monthly compounding?
- daily compounding?
- continuous compounding?

The quoted 5% rate is called the annual percentage rate. The end-of-year return on a $1 investment is the effective annual rate.

A Caveat

- Caution: This terminology is not universal.
- What is important is the economic idea:

$$\text{end of year return} = \left(1 + \frac{\text{quoted rate}}{n}\right)^n - 1.$$

- We will use the terminology as we have defined it.

In-Class Exercise

Reported interest rates are often quoted as semiannual rates (for reasons we will learn later). In the Treasury Strip example, we had the following quoted rates. Our discount rates, however, needed to be expressed in effective annual rates. Verify that the following rates are calculated correctly.

Year	Quoted Rate	Discount Rate
August 98	5.72%	5.80%
August 99	5.94%	6.03%
August 00	6.06%	6.15%
August 01	6.16%	6.25%

Example: Monthly Discount Rates

A Treasury bill is a debt contract in which the government promises to pay \$100,000 at a date that is less than one-year from the date of the promise. Investors pay a discount from the face value (the \$100,000). The return from today until the maturity of the contract can be calculated as \$100,000/(today's price). On August 28, the prices and approximate maturities of T-bills were

Maturity	Price
41 days	\$99,429
76 days	\$98,928
104 days	\$98,532

Calculate the APR for each maturity.

Application: Monthly Discount Rates

You are the treasurer of a company that has just purchased inventory at a \$300,000 invoice price. Your supplier will let you pay at a 2.5% discount today for the inventory, or you can pay \$100,000 on approximately the 15th of each of the next 3 months. Part of your job is to decide to best manage the company's cash reserves by making short-term (riskless!) investments if necessary.

Using the data in the previous example, decide whether to take the discount or to postpone payment. If you postpone payment, you will buy T-bills that mature in the number of days in the previous slide. Assume that the company has sufficient cash on hand to take either alternative.

Perpetuities: Example

Recall that the tobacco settlement calls for the tobacco industry to pay beginning in 2022 $15 billion each year forever. This is a perpetuity because its cash flows are perpetual. Also recall that the payments are stated in real dollars, so the appropriate discount rate would be the real rate of interest, which we take to be 2.5%. Recall that the tax rate for the industry is 40%. What is the after-tax present value of the perpetuity?

Perpetuities: General Formula

Suppose that a perpetuity grows at the constant rate g each year. The PV of this perpetuity at the constant discount rate r is given by

$$\frac{C_1}{r-g}$$

if $r > g$.

Annuities: Example

You have the opportunity of extending by one year the lease on your current apartment in Back Bay. Your rent would be \$1100 per month, and you would have to pay \$150 per month in parking, \$100 per month in repairs, and \$400 per month in utilities. Alternatively, you could sign a one-year lease on an apartment in Cambridge that costs \$1400 per month. Parking would be free, you would not be responsible for repairs, and utilities cost \$300 per month. The cost of moving would be \$500 payable immediately. Should you switch apartments? Calculate the value of doing so if the relevant discount rate is 5% (this is an EAR). Assume that it is now the end of September.

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Annuity Formula

An annuity is a stream of constant cash flows over a fixed period of time.

The formula for the present value of an annuity that pays C for t periods is given by

$$C\left[\frac{1}{r}-\frac{1}{r(1+r)^t}\right]$$

where r is the relevant rate.

To derive this expression, we can use the fact that an annuity can be viewed as the difference of two perpetuities.

The term in brackets is called an annuity factor.

Conclusion

We have learned a lot in this lecture. You will be well-positioned for the next lecture if you understand the following:

1. Money has time value.

2. Whether or not a project should be accepted depends on its NPV.

3. The appropriate discount rate is your opportunity cost of capital.

4. Dealing with uncertainty requires considering risk aversion.

5. Compounding is important, and the EAR can be very different depending on the number of compounding periods.

6. The formulas and the corresponding intuition.