

15.415 Finance Theory

Lecture 4: Valuation of
Fixed Income Securities I

Spring 1999

Overview of this Lecture

In this lecture, we study the following:

- What are bonds? What are their properties?
- What are some examples of bonds? How do we find their prices from sources of financial data?
- What is the term structure of yields? How do we find it?

Answers to these questions use the tools we developed in the previous lectures.

Some of the material is drawn from:

Suresh Sundaresan, *Fixed Income Markets and Their Derivatives*, Southwestern College Publishing.

What is a Bond?

A bond is a legal contract in which a borrower promises to pay a specified stream of cash flows in exchange for some consideration (such as cash, securities, or fixed assets).

- Typical bond: Pays fixed semiannual coupons during its life and then pays its par value at maturity.
 - The par value is the face value of the bond. Typically this amount is paid at the maturity of the bond.
 - The coupon rate is the bond's interest payments per dollar of par value. Coupons are typically paid semiannually.
 - The time to maturity is the length of time that the bond is outstanding.

- Example: Describe the features of the following Treasury bond.

$5\frac{3}{4}$ Dec 98

- The price of a bond depends on its coupon, its time to maturity, its par value, and investors' required rate of return for bonds with similar features.
- Prices of bonds are typically quoted in one of two ways:
 1. Percentage of par
 2. Yield to maturity – IRR of bond

By convention, YTM is an APR

YTM is popular for quotes

Zero-Coupon Bonds

A zero-coupon bond is a bond that pays no coupons. The owner of a zero-coupon bond receives a return through the appreciation of the bond's price.

Examples of zero-coupon bonds:

- Treasury Bills – short-term debt instruments issued by the U.S. government to finance government spending. By definition, their maturity at issuance is (approximately) less than a year, and they pay no coupons.
- Separate Trading of Registered Interest and Principal Securities (STRIPS) – longer term zero-coupon bonds created by investment banking firms who buy coupon-paying Treasury bonds and sell rights to single payments backed by the bonds.

Treasury Bills

- Par value typically equals \$1,000,000.
- The prices of T-bills are quoted in terms of bank discounts.

Example: On Sept 8, 1997 (WSJ), we see

Maturity	Days to Maturity	Bid	Asked	Change	Asked Yield
Dec 26 '97	108	5.05	5.03	...	5.18

- For a T-bill with n days to maturity

$$\text{bank discount} = \frac{360}{n}(100 - \text{Price}),$$

where the price is a percentage of par.

Find the bid price and asked price for this T-bill.

YTM for T-Bills

The YTM for Treasury bills with less than 182 days to maturity is

$$\text{YTM} = \left(\frac{100}{\text{Price}} - 1 \right) \times \frac{365}{n}$$

For T-bills with more than 182 days to maturity, the convention is to assume that the interest is paid after the 182 days and then is reinvested for the remaining time to maturity.

This is also known as a bond equivalent yield.

Graph of price change to maturity assuming constant yield.

Graph of price change as yield changes assuming constant maturity.

Some Market Microstructure

- The bid price is the price at which a dealer is willing to buy; the asked price is the price at which a dealer is willing to sell. Typical spread for a T-bill: 1-2 basis points.
- The difference between the asked price and the bid price is the bid-asked spread. This is a type of transaction cost. Why must the spread always be nonnegative?
- There are many determinants of bid-asked spread: competition among dealers, dealer inventory, information asymmetry,...
- Bid-asked spreads are usually positive, but we will sometimes assume that transactions costs are zero to simplify things. Sometimes generalizations are not easy...

Treasury Strips

Example: On September 8, 1997 (WSJ), we see

Maturity	Type	Bid	Asked	Change	Ask Yld
Aug 99	ci	89:09	89:11	+1	5.93
Aug 99	np	89:08	89:09	+1	5.96

- What does this mean?
- YTM's for strips are computed using semiannual compounding.

Find the selling price, buying price, and YTM on the principal-only strip above.

Graph of term structure today.

Graph of term structures in BKM

Term Structure of Interest Rates

The relation between the interest rates and time to maturity of bonds that differ only in their time to maturity is known as a term structure of interest rates.

- Example: The term structures in the previous 2 slides are known as the term structure of spot rates.

An n -period spot rate is the yield to maturity on a zero-coupon bond that matures n periods from today.

- Term structures can have many shapes – spot rates vary with maturity.
- In the future, we will sometimes assume that the term structure is flat, meaning that spot rates are constant.

Generalization is sometimes easy; sometimes it requires advanced mathematics.

Forward Rates: Example

Suppose that the annually compounded 2-year spot rate is 5.80% and the annually compounded 3-year spot rate is 6.00%. What implicit rate prevails between years 2 and 3?

In-Class Exercise

Suppose that the annually compounded 1-year spot rate is 5.75% and the annually compounded 3-year spot rate is 6%. What is the implied rate of interest per annum for the two-year period beginning one year from today?

Is it theoretically possible to borrow and lend at this implicit rate? Assume that you can borrow and lend at the spot rates.

Forward Rates

Spot rates are observable today. Forward rates are interest rates that are implied by the current term structure of spot rates.

Let r_n be a n -year spot rate. The forward rate that holds from year n to year $m + n$ is given by

$$(1 + {}_n f_m)^m = \frac{(1 + r_{m+n})^{m+n}}{(1 + r_n)^n}$$

In the preceding examples

- ${}_2 f_1 =$

- ${}_1 f_2 =$

Coupon Bonds

Coupon bonds are bonds that pay a nonzero coupon.

- The price of a coupon bond is given by

$$\text{Price} = \sum_{j=0}^{n-1} \frac{C}{\left(1 + \frac{y}{2}\right)^{j + \frac{z}{x}}} + \frac{100}{\left(1 + \frac{y}{2}\right)^{n-1 + \frac{z}{x}}}$$

where n = number of coupon dates until maturity, z = number of days until next coupon date, x = total number of days between last and next coupon dates, and C = coupon payment as % of par.

- Recall that the bonds yield to maturity is the IRR of the bond. Note that it is semiannually compounded.
- A coupon bond can sell above or below its par value. A bond that sells above par sells at a premium; otherwise, it sells at a discount (but it is not a discount bond!).

In-Class Exercise

A bond has 5 years and 94 days to maturity. It pays a semiannual coupon of 6%. The bond pays its next coupon 94 days from today; there are 184 days between the last coupon payment and the next coupon payment.

1. Calculate the price of the bond assuming that its yield to maturity is 8%.
2. Calculate the price of the bond assuming that its yield to maturity is 4%.

Graph of previous bond as a function of y_{tm}

Graph of previous bond (20 year mat) as a function of ytm

Treasury Notes and Bonds

Treasury notes and bonds are debt obligations of the U.S. government.

- A T-note has a maturity of between 1 and 10 years.
- A T-bond has a maturity of greater than 10 years when issued; most have maturities less than 30 years. The 30-year bond is often called the long bond.
- T-notes and T-bonds typically pay semiannual coupons on the 15th of the month. The bond also matures on the 15th.
- Prices are usually quoted in $1/32$ s.
- Spreads on the most traded bonds are usually $1/32$ or $2/32$.

Quoted Prices for T-notes

Consider the following price of a T-note from the WSJ (Sept 8, 1997):

Rate	Maturity	Bid	Asked	Change	Yield
$5\frac{1}{4}$	Jul 98n	99:18	99:20	+1	5.68

- The invoice price (full price, dirty price) is the price that the buyer of a bond must pay. This price is the present value of the coupon payments and par value of the bond.

Quoted prices of bonds are often not invoice prices!

- The difference between the quoted price (flat price, clean price) and the invoice price is the amount of accrued interest.

Accrued interest is computed using linear amortization.

- The quoted yields may not be yields to maturity if the bond sells above par! In that case, it may be a yield to call. (We ignore this for now.)

Dealing with Accrued Interest

In the T-note example above:

- The flat ask price is 99:20. The bond pays coupons on January 15 and July 15.
- There are 129 days from September 8, 1997, to January 15, 1998. There are 184 days from July 15, 1997, to January 15, 1998. The accrued interest is

$$\frac{184 - 128}{184} \times \frac{5\frac{1}{4}}{2} = 0.7989$$

- The invoice price is

$$99:20 + 0.7989 = 99.6250 + 0.7989 = 100.4239 \\ \approx 100:14$$

- The yield to maturity y satisfies:

$$100:14 = \left(2.625 + \frac{102.625}{1 + \frac{y}{2}} \right) / \left(1 + \frac{y}{2} \right)^{128/184}$$

It equals 5.68%.