

Solution to Problem Set 1

1. Investment 1 has an NPV of

$$-10000 + \frac{20000}{1 + 20\%} = 6667.$$

Similarly, investments 2, 3, and 4 have NPV's of 5000, -417, and 2167, respectively. The (internal) rate of return on investment 1 is defined by

$$(1 + r) \times 10000 = 20000 \Rightarrow r = 100\%.$$

Similarly, investments 2, 3, and 4 have rates of return of 140%, 10%, and 150%, respectively.

- (a) The most valuable investment is 1, since it has the highest NPV.
 - (b) Investment 1 should be undertaken, because it has the highest NPV.
2. (a) First, from the effective annual rate, we get the monthly rate, x :

$$x = (1 + 0.08)^{\frac{1}{12}} - 1 \Rightarrow x = .6434\%.$$

The APR is $.6434\% \times 12 = 7.7208\%$. The total payment that needs to be made other than the down payment is $300,000 - 15,000 = 285,000$. It should be paid in $30 \times 12 = 360$ months. This is an annuity problem. The monthly payment is

$$\frac{285000}{\frac{7.7208\%}{12}} \times \left(1 - \frac{1}{(1 + \frac{7.7208\%}{12})^{360}}\right) = 2036.$$

The amortization table is table 1. The interest is equal to the outstanding principal times $7.7208\%/12$.

- (b) Because your interest payments on the mortgage are tax-deductible, you get tax credits which are treated as cash inflows. For the next three years, the tax credits are:
 - On 04/01/98, get credits for the interest paid during 11/01/97 to 12/01/98 (month 1 to month 2) sum = 3666, tax credit = 1027.
 - On 04/01/99, get credits for the interest paid during 01/01/98 to 12/01/99 (month 3 to month 14) sum = 21884, tax credit = 6128.

- On 04/01/2000, get credits for the interest paid during 01/01/99 to 09/01/99 (month 15 to month 23) sum = 16280, tax credit = 4558.

The cash flows from buying the apartment are summarized in table 2. Note the following. First, the selling price is $320000 \times (1 - 5\%) = 304000$. Second, the PV of the mortgage is computed using the annuity formula

$$\frac{2036}{\frac{7.7208\%}{12}} \times \left(1 - \frac{1}{\left(1 + \frac{7.7208\%}{12}\right)^{23}}\right) = 43399.$$

Third, the closing payment on the mortgage is the outstanding principal on September 1, 1999, plus the one month interest, i.e.

$$280002 \times \left(1 + \frac{7.7208\%}{12}\right) = 281804.$$

Month	Monthly Payment	Outstanding Principal	Interest	Principal Reduction	Outstanding Principal After Payment
1	2036	285000.00	1833.69	202.31	284797.69
2	2036	284797.69	1832.39	203.61	284594.08
3	2036	284594.08	1831.08	204.92	284389.16
4	2036	284389.16	1829.76	206.24	284182.92
5	2036	284182.92	1828.43	207.57	283975.35
6	2036	283975.35	1827.10	208.90	283766.45
7	2036	283766.45	1825.75	210.25	283556.20
8	2036	283556.20	1824.40	211.60	283344.60
9	2036	283344.60	1823.04	212.96	283131.64
10	2036	283131.64	1821.67	214.33	282917.31
11	2036	282917.31	1820.29	215.71	282701.60
12	2036	282701.60	1818.90	217.10	282484.50
13	2036	282484.50	1817.51	218.49	282266.01
14	2036	282266.01	1816.10	219.90	282046.11
15	2036	282046.11	1814.68	221.32	281824.79
16	2036	281824.79	1813.26	222.74	281602.05
17	2036	281602.05	1811.83	224.17	281377.88
18	2036	281377.88	1810.39	225.61	281152.26
19	2036	281152.26	1808.93	227.07	280925.20
20	2036	280925.20	1807.47	228.53	280696.67
21	2036	280696.67	1806.00	230.00	280466.67
22	2036	280466.67	1804.52	231.48	280235.20
23	2036	280235.20	1803.03	232.97	280002.23

Table 1: The Amortization Table

description	time	cash flow	discount factor	NPV
down payment	10/01/97	-15000	1	-15000
selling house	10/01/99	304000	$\frac{1}{(1 + \frac{7.7208\%}{12})^{24}}$	260631
close mortgage	10/01/99	-281804	$\frac{1}{(1 + \frac{7.7208\%}{12})^{24}}$	-241602
tax credit	4/01/98	1027	$\frac{1}{(1 + \frac{7.7208\%}{12})^6}$	988
tax credit	4/01/99	6128	$\frac{1}{(1 + \frac{7.7208\%}{12})^{18}}$	5460
tax credit	4/01/00	4558	$\frac{1}{(1 + \frac{7.7208\%}{12})^{30}}$	3760
mortgage	11/97-9/99	-2036/mo		-43399
total				-29162

Table 2: Cash Flows from Buying

If you rent, the NPV is

$$-\frac{1800(1 + \frac{7.7208\%}{12})}{\frac{7.7208\%}{12}} \left(1 - \frac{1}{(1 + \frac{7.7208\%}{12})^{24}} \right) = -40168.$$

Note here the annuity formula needs to be modified: instead of getting the first payment at the *end* of the first period, we are getting it at the *beginning*. There are two ways to modify it:

- You could treat the total cash flow as the sum of the first monthly payment (discount factor is 1) and an annuity of 23 months.
- You could treat the total cash flow as an annuity. By doing that you are effectively postponing each payment for one month, so after getting the result, you need to multiply by $(1 + \text{monthly rate})$ to get back the correct figure.

I am using the second method. You should check and verify that you get the same result by the first method.

Our NPV analysis shows that it is better to buy.

- (c) The rent R that makes you indifferent between renting and buying is defined by

$$\frac{R(1 + \frac{7.7208\%}{12})}{\frac{7.7208\%}{12}} \left(1 - \frac{1}{(1 + \frac{7.7208\%}{12})^{24}} \right) = 29162 \Rightarrow R = 1307.$$

3. (a) There are two reasons:

- The PV of the payment that the viatical insurance company receives when the patient dies decreases.
- More monthly payments need to be made by the viatical insurance company.

- (b) Denote the monthly rate by x . If the patient lives for one year, the present value of the payments received by the viatical insurance company is

$$-15000 - \frac{75(1+x)}{x} \left(1 - \frac{1}{(1+x)^{12}} \right) + \frac{20000}{(1+x)^{12}}.$$

This present value must be 0. Solving this non-linear equation numerically (with Solver (on Excel) for instance) we get $x = 1.9784\%$. The APR is 23.74% and the EAR is 26.50%.

If the patient lives for two years the equation becomes

$$-15000 - \frac{75(1+x)}{x} \left(1 - \frac{1}{(1+x)^{24}} \right) + \frac{20000}{(1+x)^{24}} = 0.$$

We now get $x = 0.7665\%$. The APR is 9.20% and the EAR is 9.60%.

- (c) The monthly rate is: $(1 + 15\%)^{\frac{1}{12}} - 1 = 1.1715\%$. The company is willing to pay

$$\frac{50000}{(1 + 1.1715\%)^{12}} - \frac{200(1 + 1.1715\%)}{1.1715\%} \left(1 - \frac{1}{(1 + 1.1715\%)^{12}} \right) = 41225.$$

4. (a) Probably Crosby, Stills & Nash, because they have a more predictable cash flow.
 (b) This is an annuity problem. If C is the yearly cash flow, the present value is

$$\frac{C}{7.5\%} \left(1 - \frac{1}{(1 + 7.5\%)^{10}} \right).$$

Since this present value has to be 100M, C is 14.57M.

- (c) The present value is simply $15/7\% = 214.29\text{M}$.
 5. (a) The yearly contribution is $2000 \times (1 - 28\%) = 1440$. Since you pay tax on the interest income, the relevant interest rate is $6\% \times (1 - 28\%) = 4.32\%$.

To compute the money that you have at your retirement, you can use the future value formula with 30 cash flows. You can do this in a spreadsheet. However, there is a simpler way. You can compute the present value of the cash flows, using the annuity formula, and then compute the future value of this present value, multiplying by $(1 + 4.32\%)^{30}$. The future value is

$$\frac{1440}{4.32\%} \left(1 - \frac{1}{(1 + 4.32\%)^{30}} \right) (1 + 4.32\%)^{30} = 85218.$$

- (b) The yearly contribution is the same as in the first part. However, the interest rate is 6%. The before-tax money that you have at retirement is

$$\frac{1440}{6\%} \left(1 - \frac{1}{(1 + 6\%)^{30}} \right) (1 + 6\%)^{30} = 113844.$$

You pay tax on the interest income. The interest income is the difference between the 113844 and the money you would have had if the interest rate was 0%. Therefore, the interest income is $113844 - 30 \times 1440 = 70644$ and the tax is $70644 \times 0.28 = 19780$. Your retirement money is $113844 - 19780 = 94064$.

- (c) The yearly contribution is 2000 and the interest rate is 6%. The before-tax money at retirement is

$$\frac{2000}{6\%} \left(1 - \frac{1}{(1 + 6\%)^{30}} \right) (1 + 6\%)^{30} = 158116.$$

You retirement money is $158116 \times (1 - 28\%) = 113844$.

- (d) The benefit should increase, because the deferred tax on which interest accrues is greater.
6. (a) The cash flow table is

	Cost	Revenue	Net Cash Flow
Year 0	4.6	2.52	-2.08
Year 1	0.7	0.88	0.18
Year 2	2.8	3.44	0.64
Year 3	3.1	3.84	0.74
Year 4	2.5	3.12	0.62
Year 5	1.7	2.58	0.88
Year 6	0	0.42	0.42

The revenues are computed as follows

- Year 0: $16.8 \times 15\% = 2.52$.
- Year 1: $1.1 \times (1 - 0.2) = 0.88$.
- Year 2: $4.3 \times (1 - 0.2) = 3.44$.
- Year 3: $4.8 \times (1 - 0.2) = 3.84$.
- Year 4: $3.9 \times (1 - 0.2) = 3.12$.
- Year 5: $2.7 \times (1 - 0.2) + 16.8 \times 5\% \times 0.5 = 2.58$.
- Year 6: $16.8 \times 5\% \times 0.5 = 0.42$.

The NPV is 223700. Since this is a positive NPV project, the company should take the project.

- (b) The IRR is 15.33%. Since it is greater than 12%, the company should take the project. The payback is 4 years and the discounted payback is 5 years. Since, these are smaller or equal than 5 years, the company should take the project.
- (c) The cash flow with the new payment schedule is

	cost	revenue	net cash flow
year 0	3.6	2.52	-1.08
year 1	1.83	0.88	-0.95
year 2	2.8	3.44	0.64
year 3	3.1	3.84	0.74
year 4	2.5	3.12	0.62
year 5	1.7	2.58	0.88
year 6	0	0.42	0.42

The year 0 cost is now 3.6M and the year 1 cost is $0.7+1.13=1.83$ M. The NPV is 214800, so the original payment plan should be taken. The IRR is 15.71%. If we base our decision on IRR, we should take the new payment plan. However, this is the wrong decision.