M.I.T. **Spring 1999**
Sloan School of Management **15.415**

Problem Set 2

Due Friday, February 26

1. **Forward and Spot Rates**

   Which of the following statements are correct? You do not need to explain your answer. To get credit, you need to (i) point all the correct statements and (ii) not point any incorrect statements.

   With today's US Treasury Yield Curve, you can compute exactly:

   (a) The price at which a 5-year STRIP with $\$1,000$ face value will trade in two years

   (b) The spot rates that will prevail in two years

   (c) The price at which a 5-year T-bond with 7% coupon and $\$1,000$ face value trades today

   (d) The forward rates that prevail today

   (e) The forward rates that will prevail in two years

2. **Bond Duration**

   The yield curve is flat at 8%. Suppose you ranked the following bonds and portfolios by increasing durations. That is: 1st the bond with the shortest duration, 2nd a bond with a longer duration, 3rd a bond with an even longer duration, etc. Which one would come third? Explain why.

   (a) 19-year T-bond with coupon 10% and face value $\$10,000$

   (b) 19-year STRIP with face value $\$10,000$

   (c) 20-year T-bond with coupon 10% and face value $\$1,000$

   (d) 20-year STRIP with face value $\$100,000$

   (e) A portfolio with $\$1,000$ invested in Bond B and $\$12,000$ invested in Bond C.

3. **Bond Prices**

   The yield curve is flat at 8%. Suppose you ranked the following US T-bonds, all with fact value $\$1,000$, by increasing prices. That is: 1st the bond with the lowest price, 2nd a bond with a higher price, etc. Which bond would come in fourth position? Explain why.

(a) 20-year bond with 7% coupon

(b) 20-year bond with 9% coupon

(c) 5-year bond with 7% coupon

(d) 5-year bond with 8% coupon

(e) 5-year bond with a 9% coupon

4. **Synthetic Bonds**

Assume that you are the manager of a large portfolio of Treasury securities, including bonds, notes, bills, and strips. You are studying whether it is worthwhile to replace some of the Treasury notes in your portfolio with "synthetic bonds" that replicate the cashflows of these Treasury notes. If the synthetic bonds are cheaper than the T-notes, then you can sell the T-notes and buy the synthetic bonds for an immediate profit.[1]

Here are the September 8, 1997 prices of two Treasury notes that are in your portfolio (WSJ, September 9, 1997)

Treasury notes

Rate	Maturity Mo/Yr	Bid	Asked
6	Aug 99	100:00	100:02
8	Aug 99	103:20	103:22

Here are the September 8, 1997 quoted prices of four Treasury Strips that will be useful for constructing the synthetic bonds

Treasury Strips (Principal Only)

Mat.	Bid	Asked
Feb 98	97:20	97:20
Aug 98	94:28	94:28
Feb 99	92:01	92:02
Aug 99	89:08	89:09

(a) Explain how you would use the strips to construct a synthetic bond that replicates a given T-note. Do not worry that the replicating strategy might require you to buy only a fraction of a strip (i.e., assume that if it necessary you can, for example, buy 0.51283 Feb 98 strips to replicate the bond).

(b) Compare the bid prices of the notes to the ask prices of the strips to simulate selling the notes and buying the synthetic bond. Do you make a profit?

Do not forget to adjust for accrued interest. There are 184 days from August 15, 1997 to February 15, 1998. The accrued interest is for the 25 day period

[1]This is an example of an arbitrage opportunity. An arbitrage opportunity is a collection of trades that requires no net investment and provides a profit with no chance of loss. One of the most powerful results in modern finance theory is that arbitrage opportunities cannot persist in a well-functioning market possessing at least one greedy trader. By repeatedly executing the arbitrage, the trader would force prices to realign until the arbitrage disappears.

from August 16, 1997 to September 9, 1997. (Although you sell the notes on September 8, you keep them until September 9 because of the 2-day settlement.) Since accrued interest is computed using linear amortization, it is 25/184 times the coupon payment.

(c) Suppose that you own the strips that would be necessary to replicate the T-note. Can you make a profit by buying the notes and selling the synthetic bonds?

(d) Repeat questions 1b and 1c, using instead the August 2002 Treasury Note with a 6 3/8 coupon. Use the quoted prices from the WSJ cutout included with this problem set. Use the Principal Only, i.e. `np`, strips. Note that the WSJ does not have a price for the February 2002 `np` strip. This is probably because there were no transactions on the strip that day. Use the price for the Coupon Interest, i.e. `ci`, strip instead.

5. **Bootstrapping**

In class, we determined the term structure of interest rates from strips. Since strips are zero-coupon bonds, their yields give us directly the spot rates. However, strips are not as liquid as Treasury notes or bonds, a fact which may lead us to misleading interpretations of the term structure. As a result, it is common to determine the term structure from T-notes or bonds, using a procedure called bootstrapping. This procedure is less direct, but may give more accurate answers.

Suppose that today is August 18, 1997, and you want to use bootstrapping to determine the spot rates for the next 6 months, the next year, the next 18 months, and so on, until the next 5 years. You first find the T-bills that mature 6 months and one year from today. The closest are the February 19, 1998 bill and the August 20, 1998 bill. The quoted prices are (WSJ, August 19, 1997)

Treasury Bills

Maturity	Days to Mat.	Bid	Asked
Feb 19 '98	184	5.13	5.12
Aug 20 '98	366	5.20	5.19

You next find T-notes that mature 18 months until 5 years from today. The quoted prices are

Treasury notes

Rate	Maturity Mo/Yr	Bid	Asked
5	Feb 99	98:28	98:30
8	Aug 99	104:00	104:02
8 1/2	Feb 00	105:29	105:31
6	Aug 00	100:05	100:06
5 5/8	Feb 01	98:24	98:26
6 1/2	Aug 01	101:17	101:19
6 1/4	Feb 02	100:19	100:21
6 3/8	Aug 02	101:07	101:09

(a) You decide to simplify your analysis by using the average of the bid and the asked price as the price at which you can buy or sell a bond. In other words, you ignore transaction costs. Verify that the average prices are

Average of Bid and Asked Price	
Maturity	Price
Feb 19 '98	97.38
Aug 20 '98	94.72
Feb 99	98.96
Aug 99	104.12
Feb 00	106.03
Aug 00	100.24
Feb 01	98.84
Aug 01	101.63
Feb 02	100.69
Aug 02	101.32

For the T-notes do not forget to adjust for accrued interest. There are 184 days from August 15, 1997 to February 15, 1998. The accrued interest is for the 4 day period August 16, 1997 to August 19, 1997.

(b) You also decide to ignore rounding errors in the 6 month periods. In other words, you do your calculations as if you were on August 15, 1997, and the T-bills matured on February 15, 1998, and on August 15, 1998. (However, you use the prices in part (a).) Using bootstrapping, compute the spot rates from 6 months to 5 years. Express these spot rates as semiannually compounded APR's.

6. **Liability Immunization**

You are an actuary at an insurance company that sells to customers contracts that pay a given amount at some point in the future. Customers purchase these contracts at a discount and hold them until they mature. Although these contracts are essentially zero-coupon bonds issued by an insurance company, they are usually called *guaranteed investment contracts* (GIC) by the insurance industry. These contracts are often purchased by investors for their retirement accounts.

Your insurance company has been selling these contracts for several years. Six months from today, you expect the aggregate payment for the maturing GICs to be \$11 million. Thereafter, you expect payments for maturing GICs to grow forever at the rate of 0.5% every six months. Your company has always made semiannual GIC payments, and you expect this to continue in the future.

Lately, management has been concerned about adverse profitability effects due to the interest rate risk the company faces in its GIC liabilities. You have been asked to explore immunization methods to reduce the impact of interest rates on the present value of the GIC portfolio. In particular, management wants to offset such risk by making appropriate investments in government bonds.

Your analysis requires answers to several questions. For all of these questions, you are to assume that the term structure is flat, and assume that the yield on all fixed-income cash flows is 8% semiannually compounded.

(a) Explain how, in the absence of immunization, the interest rate risk of the GIC liabilities can adversely affect the insurance company's profitability. Explain why the company cannot exactly offset this risk (say, by purchasing Treasury bonds that exactly replicate the present value of the GIC liabilities). How will management's immunization plan reduce this risk?

(b) What is the present value of the company's GIC liabilities? Note that the formula given in class must be adjusted for semiannual rates.

(c) Quantify the interest-rate risk by calculating Macaulay duration, modified duration, and convexity of the GIC liabilities.

> *Hint*: Use the formula for the present value of a constant growth annuity that we discussed in class. Using some calculus, you should be able to calculate exact formulas for the modified duration and the convexity. Use the relation between modified and Macaulay duration to solve for the latter.

(d) You will consider two government bonds for your analysis. The first is a zero-coupon bond that matures in 18 years, and the second is a 10% coupon bond that also matures in 18 years. Both bonds have a par value of $1000, and any coupon payments are paid semiannually starting six months from today. Calculate the price, Macaulay duration, modified duration, and convexity for each of these bonds.

(e) Set up a portfolio of the two bonds that has the same price and approximate price changes as the GIC liabilities.

> *Hint*: Your portfolio will be represented by two numbers, say x_1 and x_2. The cost of your portfolio should equal the present value, P, of the GIC liabilities; i.e.,
>
> $$P = x_1 P_1 + x_2 P_2, \tag{1}$$
>
> where P_i is the price of bond i, $i = 1, 2$.
> In addition, you will want the price change in the portfolio to match the change in the present value of the GIC liabilities. Recall that the first-order approximation for the price change of a fixed-income portfolio is given by
>
> $$\Delta P \approx -D^* P \Delta y,$$
>
> so x_1 and x_2 satisfy
>
> $$D^* P = x_1 D_1^* P_1 + x_2 D_2^* P_2,$$
>
> where D^* is the modified duration of the GIC liabilities and D_i^* is the modified duration of bond i, $i = 1, 2$. Do not worry if x_1 and x_2 are not whole numbers: assume, if necessary, that you may purchase a fraction of a bond.

(f) Verify the approximation by graphing the price of the portfolio and the present value of the GIC liabilities as a function of the yield. For your graph, use at least the domain $\{1.5\%, 2.0\%, \ldots, 8\%, \ldots, 16\%\}$. Comment on the accuracy of the approximation.

(g) Using equation 1, derive a general formula for the modified duration and convexity of a fixed-income portfolio in terms of the modified duration and convexity of its underlying securities.