**M.I.T.** **Spring 1999**
**Sloan School of Management** **15.415**

# Problem Set 4

Due Friday, March 12

1. **Diversification**

   Only one of the following statements is correct. Which one? No explanations are needed.

   (a) Proper diversification can always eliminate systematic risk.

   (b) Proper diversification cannot always eliminate but can always at least reduce systematic risk.

   (c) The risk-reducing benefits of diversification do not occur until at least 15-20 individual securities have been purchased.

   (d) Because diversification reduces a portfolio's total risk, it necessarily reduces the portfolio's expected return.

   (e) None of the above.

2. **The CAPM**

   Security A has an expected rate of return of 13% and a beta of 1.2. The risk-free rate is 5% and the market risk premium is 6%. Which of the following statements are correct? No explanations are needed.

   (a) According to the CAPM, security A is fairly priced.

   (b) According to the CAPM, security A is overpriced.

   (c) According to the CAPM, security A is underpriced.

   (d) None of the above.

3. **Betas**

   Suppose that you short the market portfolio. What is the beta of your position? No explanations are needed.

   (a) 2

   (b) 1

   (c) 0

(d) -1

(e) Infinite

4. **True or False**

Are the following statements true or false? Give brief but precise explanations for your answers.

(a) Stock A has expected return 10% and standard deviation 15%, and stock B has expected return 12% and standard deviation 13%. Then, no investor will buy stock A.

(b) Diversification means that the equally weighted portfolio is optimal.

(c) The CAPM predicts that the expected return on the market portfolio is always greater than the return on the riskless asset.

(d) The CAPM predicts that a security with a beta of zero offers zero expected return.

(e) The CAPM predicts that all investors hold the same portfolio of risky assets.

(f) The CAPM predicts that investors demand higher expected rates of return from stocks that are very sensitive to fluctuations in the stock market.

(g) An investor who puts $10000 in T-bills and $20000 in the market portfolio will have a beta of 2.0.

5. **Calculating the Portfolio Frontier**

An Excel spreadsheet file has been placed on the course webpage. The webpage is

`http://web.mit.edu/15.415e/www`

for Professor Willard and

`http://web.mit.edu/15.415f/www`

for Professor Vayanos. The file, named `capm.txt`, is located together with this problem set. It contains the end-of-month returns from January, 1992, to December, 1996, for three companies: Caterpillar, Hewlett-Packard, and McDonald's. It also contains end-of-month returns for the S&P500 index and for the T-bill whose maturity is closest to one month. We will assume that the S&P500 represents the market portfolio and that the T-bill represents the riskless asset. Note that the returns are monthly and not yearly returns. Monthly data gives us better estimates of variance and covariance than yearly data, for reasons that need not concern us here.

In this problem, we will use these returns to construct the historical portfolio frontier for portfolios that consist of holdings in these three companies. To do this, you will need to complete several tasks. You may complete each of these tasks in any manner that you wish, including doing the optimizations by hand or doing them using modern software (Excel, Mathematica, Matlab, etc.). The use of Excel was discussed in class, and more hints are provided here. Please contact us if you do not have access to Excel or some other math software.

(a) Calculate the sample mean and sample standard deviation of the historical monthly returns for the three companies. You may find the Excel spreadsheet functions `average` and `stdev` useful for this. Also calculate the sample correlation matrix for the monthly returns of the three companies. You may find the Excel spreadsheet function `correl` useful for this. For this problem, you must express the sample means, standard deviations, and correlations accurate to 5 decimal places (i.e., each number should be correct when expressed as 0.xxxxx when the usual rounding rules are used).

(b) Use the sample means, standard deviations, and correlation matrix that you calculated in part (a) to calculate the sample mean and sample standard deviation of a portfolio that is (i) equally weighted in each of the three stocks, and (ii) weighted according to the ratio 3:2:1 in Caterpillar, Hewlett-Packard, and McDonald's, respectively. Your answers must be accurate to 5 decimal places.

(c) We now give you the sample means and standard deviations accurate to 3 decimals and the correlation matrix accurate to 2 decimal places. You are to use these numbers for the remaining calculations in the problem set. (This gives you an opportunity to get correct answers for the remaining problems even if your answers to part (a) and (b) are incorrect.)

| | Caterpillar | Hewlett-Packard | McDonald's |
|---|---|---|---|
| Mean | 0.025 | 0.026 | 0.017 |
| Standard Deviation | 0.073 | 0.088 | 0.054 |

| | Caterpillar | Hewlett-Packard | McDonald's |
|---|---|---|---|
| Caterpillar | 1.00 | 0.40 | 0.33 |
| Hewlett-Packard | | 1.00 | 0.20 |
| McDonald's | | | 1.00 |

Fill in the entries in the table below. The reported standard deviation and portfolio weights in a given row should be that of the frontier portfolio corresponding to the given mean. You can use `Solver` in Excel to solve for these portfolio weights.

(d) Plot the portfolio frontier using the entries in the preceding table. Plot each of the individual companies inside the feasible set. Comment on the weights of the portfolios along the portfolio frontier, making sure that you include some discussion of the correlation among the 3 companies.

(e) Suppose there is a riskless asset that has a monthly return of 0.6%. Plot the resulting portfolio frontier and identify the tangent portfolio. Verify that the "buck-to-bang ratio" is constant across the three companies. (You must verify this for each of the three companies.)

| Mean | Minimum Std Dev | Weight on Caterpillar | Weight on Hewlett-Packard | Weight on McDonald's |
|---|---|---|---|---|
| 3.0% | | | | |
| 2.9% | | | | |
| 2.8% | | | | |
| 2.7% | | | | |
| 2.6% | | | | |
| 2.5% | | | | |
| 2.4% | | | | |
| 2.3% | | | | |
| 2.2% | | | | |
| 2.1% | | | | |
| 2.0% | | | | |
| 1.9% | | | | |
| 1.8% | | | | |
| 1.7% | | | | |
| 1.6% | | | | |
| 1.5% | | | | |
| 1.4% | | | | |
| 1.3% | | | | |
| 1.2% | | | | |

6. **Computing Betas**

   In this problem we will use the historical returns of Caterpillar, Hewlett-Packard, and McDonald's, to estimate the betas and alphas of these companies.

   (a) Regress excess stock returns on excess market returns to obtain estimates of the stocks' betas. What is the standard deviation of these estimates?

   (b) What are the estimates of the alphas and of the standard deviation of idiosyncratic risk?

   (c) According to the CAPM, which stock should have the highest expected return? Which stock should have the lowest expected return? Estimating expected returns from sample averages, check whether these CAPM predictions are supported by the data.

**For Your Reference**

We have provided the monthly returns for 5 years on 3 stocks. These returns come from the Center for Research on Security Prices (CRSP) (pronounced "crisp") dataset. This dataset contains returns and a lot more for many more companies over the years 1926 to 1997. You may access a website provided by the Sloan Trading Room to obtain data for companies in which you have particular interest. We will provide instructions for getting data for an example company. Getting data for other companies is similar.

Using your favorite web browser, open the location `http://risk.mit.edu:8080/`

You should see several databases that are available to you. Click on "CRSP Stocks". To obtain the data, you will need the password. For this class, it is "8542*". Suppose that you are interested in getting data for Anheuser-Busch. Looking in the Wall Street Journal, you find that the ticker symbol for A-B is "BUD". Enter this where it asks for the security's CUSIP, Ticker, or Permno. Click on the beginning and ending dates for the data you want. Click on "Submit Request". In just a few seconds, you should see the data you want.

Have fun!