

Problem Set 6

Due Friday, April 23, 1999

1. Arbitrage Bound

Prove that $c^A - p^A > S - X$, where c^A and p^A are American calls that have the same exercise prices and maturity dates. Assume that the stock pays no dividends.

Note: You should prove this for general parameters – do not assume particular numbers for the stock price, exercise price, etc.

2. Binomial Option Pricing

A stock price is currently \$100. Over each of the next two 3-month periods, it will go up by 10% or down by 10%. The risk-free interest rate is 6% per annum with semiannual compounding.

- (a) What is the value of a 6-month European call option with a strike price of \$100? Calculate this directly using a two-period binomial tree and the delta-hedging approach.
- (b) What is the value of a 6-month European put option with a strike price of \$100? Calculate this directly using a two-period binomial tree and the delta-hedging approach.
- (c) Recalculate the prices using risk-neutral probabilities.
- (d) Verify that your answers satisfy put-call parity.
- (e) Find a portfolio consisting of lending or borrowing at the riskless rate and investing in the stock that replicates the payout of the European call option.

3. Black-Scholes Option Pricing

Assume that a stock price follows a geometric Brownian motion with a drift of 15% and a volatility of 30%. The current stock price is \$50.

- (a) What is the probability distribution of the log of the stock price in six months?
- (b) What is the probability that a European call option with an exercise price of \$49 and six months to maturity will be exercised?
- (c) What is the price of the European call if the riskless rate is 5.5% continuously compounded?