Solution to Problem Set 3

1. We have

$$P_0 = \frac{D_1}{r-g} = \frac{p(EPS_1)}{r-g} = \frac{p(EPS_0)(1+g)}{r-g}.$$

Using $p = 45\%$, $g = 10\%$, and $r = 15\%$, we get

$$\frac{P_0}{EPS_0} = 9.9.$$

2. The dividend growth rate is given by

$$g = ROE \times (1-p) = 9\%.$$

The price is given by

$$P_0 = \frac{1}{(1+r)^2}\frac{D_3}{r-g} = \$53.15.$$

3. (a) The dividend growth rate is given by

$$g = ROE \times (1-p) = 10\%.$$

The market capitalization rate is given by

$$r = g + \frac{D_1}{P_0} = g + \frac{p(EPS_1)}{P_0} = 20\%.$$

(b) The price is given by

$$P_0 = \frac{D_1}{r-g} = \frac{p(EPS_1)}{r-g}.$$

If $p = 1$, $g = 0$, and $P_0 = \$10$. The price does not change. This is because the return on equity is the same as the market capitalization rate. The company does not add value by investing.

(c) The price would not change for the same reason as before. To check this mathematically, note that for $p = 25\%$, $g = 15\%$, and $P_0 = \$10$.

(d) Since the stock pays no dividends, the price would become 0.

4. (a) We have

$$P_0 = \frac{D_1}{r - g}.$$

Therefore

$$g = r - \frac{D_1}{P_0} = 9.46\%.$$

(b) Using the P/E ratio, we can find earnings per share.

$$EPS_0 = \frac{P_0}{P/E} = 9.45.$$

The dividend payout ratio is thus

$$p = \frac{D_1}{EPS_1} = \frac{D_1}{(EPS_0)(1+g)} = 23.20\%,$$

and the ROE is

$$ROE = \frac{g}{1-p} = 12.32\%.$$

5. (a) Summing the changes in the DJIA stocks gives 17.625. The change in the index is then

$$\frac{17.625}{0.25450704} = 69.25152247.$$

Accurate to four decimals, the change in the index is 69.2515. Note that this is consistent with the change reported by the press on that day.

(b) At the end of 1992, the sum of the prices of the 3 companies is

$$\$18.25 + \$14.50 + \$34.00 = \$66.75.$$

For the index value to be 100 at the end of 1992, we would choose the divisor to be \$0.6675.

At the end of 1993, the sum of the prices of the 3 companies is

$$\$18.75 + \$13.50 + \$30.00 = \$62.25.$$

If the divisor is constant, the value of the index at the end of 1993 is

$$\frac{62.25}{0.6675} = 93.2584.$$

The change in the index then equals 93.2584 -100 = - 6.7416.

At the end of 1994, the sum of the prices of the 3 companies is

$$\$13.50 + \$10.00 + \$39.50 = \$63.00,$$

so the value of the index at the end of 1994 is

$$\frac{63.00}{0.6675} = 94.3820.$$

The change in the index equals 94.3820 - 93.2584 = +1.1236.
At the end of 1995, the sum of the prices of the 3 companies is

$$\$23.50 + \$9.00 + \$28.25 = \$60.75,$$

so the value of the index at the end of 1995 is

$$\frac{60.75}{0.6675} = 91.0112.$$

The change in the index then equals 91.0112 - 94.3820 = -3.3708.
At the end of 1996, the sum of the prices of the 3 companies is

$$\$25.50 + \$9.25 + \$42.00 = \$76.75,$$

so the value of the index at the end of 1996 is

$$\frac{76.75}{0.6675} = 114.9813.$$

The change in the index then equals 114.9813 - 91.0112 = 23.9701.

(c) At the end of 1992, the sum of the market values of the 3 companies is

$$(36.87)(\$18.25) + (5.78)(\$14.50) + (89.36)(\$34.00) = i\$3794.9275.$$

Note that these numbers are expressed in millions of dollars. For the index value to be 100 at the end of 1992, we would choose the divisor to be \$37.949275.
At the end of 1993, the sum of the market values of the 3 companies is

$$(37.94)(\$18.75) + (5.78)(\$13.50) + (83.69)(\$30.00) = \$3300.1050,$$

so the value of the index at the end of 1993 is

$$\frac{3300.1050}{37.949275} = 86.9609.$$

The change in the index then equals 86.9609 - 100 = -13.0391.
At the end of 1994, the sum of the market values of the 3 companies is

$$(37.95)(\$13.50) + (5.78)(\$10.00) + (80.94)(\$39.50) = \$3767.2550,$$

so the value of the index at the end of 1994 is

$$\frac{3767.2550}{37.949275} = 99.2708.$$

The change in the index then equals 99.2708 - 86.9609 = 12.3099.
At the end of 1995, the sum of the market values of the 3 companies is

$$(37.96)(\$23.50) + (5.78)(\$9.00) + (74.80)(\$28.25) = \$3057.1800,$$

so the value of the index at the end of 1995 is

$$\frac{3057.1800}{37.949275} = 80.5596.$$

The change in the index then equals 80.4496 - 99.2708 = -18.7112.
At the end of 1996, the sum of the market values of the 3 companies is

$$(38.00)(\$25.50) + (5.78)(\$9.25) + (55.84)(\$42.00) = \$3367.7450,$$

so the value of the index at the end of 1996 is

$$\frac{3367.7450}{37.949275} = 88.7433.$$

The change in the index then equals 88.7433 - 80.5596 = 8.1837.

(d) See parts (b) and (c) for the calculations. The price-weighted index "outperformed" the value-weighted index in this instance because, although the market price of Reebok's stock rose, the market value of Reebok declined substantially.

(e) The return on either index is

$$\frac{\text{Change in index} + \text{Dividends paid}}{\text{Previous index value}}.$$

The amount of dividends paid for the price-weighted index is the sum of the dividends per share in each year divided by the divisor for the price-weighted index. (We can think of this as owning 1/divisor number of shares in each company.) The amount of dividends paid for the value-weighted index is the dividends per share in each times the number of shares outstanding in that year divided by the divisor for the value-weighted index. (We can think of this as owning 1/divisor of the total market value of the companies.) These calculations are given below.

Change in PW-Index	Change in VW-Index	Dividends Paid (PW)	Dividends Paid (VW)	Return on PW-Index	Return on VW-Index
-6.7416	-13.0391	$\frac{\$1.00}{\$0.6675}$	$\frac{\$35.5850}{\$37.949275}$	-5.2434%	-12.1014%
+1.1236	+12.3099	$\frac{\$1.00}{\$0.6675}$	$\frac{\$34.7620}{\$37.949275}$	+2.8112%	+15.2090%
-3.3708	-18.7112	$\frac{\$0.90}{\$0.6675}$	$\frac{\$29.1260}{\$37.949275}$	-2.1429%	-18.0755%
+23.9701	+8.1837	$\frac{\$0.71}{\$0.6675}$	$\frac{\$18.8396}{\$37.949275}$	+27.5062%	10.7748%

The first return for the price-weighted index is

$$\frac{-6.7416 + \frac{\$1.00}{\$0.6675}}{100},$$

and the first return for the value-weighted index is

$$\frac{-13.0391 + \frac{\$35.5850}{\$37.949275}}{100}.$$

The other returns are computed similarly.

(f) The sample mean and the sample standard deviation for the return on the price-weighted index are 5.7328% and 14.8898%, respectively. The sample mean and the sample standard deviation for the return on the value-weighted index are -1.0483% and 16.4942%, respectively.

6. (a) The means and standard deviations are in table 1. The portfolio frontier is the line with the triangles.

(b) The means and standard deviations are in table 2. The portfolio frontier is the line with the squares.

(c) If the correlation is 0.3, companies A and B do not move together very much. Therefore a portfolio with positive weights on A and B can have smaller variance than both A and B. (The variance is smaller if the weight on A is 60% or above.) If the correlation is 0.7, the gains to diversification are smaller, and the minimum variance (among all portfolios that put positive weights on both stocks) is achieved by holding 100% A.