M.I.T. **Spring 1999**
Sloan School of Management **15.415**

Solution to Problem Set 4

1. The correct statement is (e). Systematic risk cannot be eliminated or reduced. Benefits to diversification occur even with two securities. Diversification does not have to lower expected return. For instance, a portfolio of two securities with equal expected returns has the same expected return as the securities, but lower variance.

2. The correct answer is (c). According to the CAPM, the expected return on the security is
$$E(R) = 5\% + 1.2 \times 6\% = 12.2\%.$$
This is lower than the actual expected return. This means that the actual price of the security is lower than the price implied by the CAPM.

3. The correct answer is (d). The return on your position is $-R_m$. Therefore, the beta is
$$-\beta(R_m) = -1.$$

4. (a) The statement is FALSE. Suppose that stocks A and B are independent. Then a 50/50 portfolio of A and B has expected return
$$0.5 \times 10\% + 0.5 \times 12\% = 11\%$$
and standard deviation
$$\sqrt{0.5^2 \times (15\%)^2 + 0.5^2 \times (13\%)^2} = 9.92\%.$$
An investor who wants to sacrifice expected return in order to reduce standard deviation, may thus hold the 50/50 portfolio.

   (b) The statement is FALSE. The equally weighted portfolio is generally not on the portfolio frontier. In the international diversification example studied in class, for instance, the equally weighted portfolio of US/Germany/Japan was inside the frontier. The reason was that the three countries have different risk characteristics and there is no reason to treat them symmetrically.

   (c) The statement is TRUE. The CAPM implies that the market portfolio is the same as the tangent portfolio. Since the expected return on the tangent portfolio is higher than the return on the riskless asset, the same is true for the market portfolio.

(d) The statement is FALSE. The CAPM predicts that the expected return on a zero beta asset is the same as the return on the riskless asset, which is generally not zero.

(e) The statement is TRUE. The CAPM predicts that investors hold a combination of the riskless asset and the market portfolio (which is the same as the tangent portfolio).

(f) The statement is TRUE. Stocks that go up a lot when the market goes up have high betas, and according to the CAPM, should have high expected returns.

(g) The statement is FALSE. The investor's return is

$$\frac{1}{3}R_f + \frac{2}{3}R_m.$$

Therefore his beta is

$$\frac{1}{3}\beta(R_f) + \frac{2}{3}\beta(R_m) = \frac{2}{3}.$$

5. (a) Using the Excel commands, we find

	Sample Means	Sample St. Devs.
Caterpillar	0.02454	0.07280
HP	0.02596	0.08822
McDonald's	0.01665	0.05381

and

Sample Correlations

	Cat	HP	McD
Cat	1.00000	0.39727	0.33444
HP		1.00000	0.20211
McD			1.00000

Our statistical analysis shows that Caterpillar and HP are similar in terms of mean and standard deviation, while McDonald's has a lower mean and standard deviation. The highest correlation is between Caterpillar and HP.

(b) To compute the mean, we use the formula

$$E(R) = w_C E(R_C) + w_{HP} E(R_{HP}) + w_M E(R_M).$$

If weights are equal to 1/3, the mean is 0.02239. If $w_C = 1/2$, $w_{HP} = 1/3$, and $w_M = 1/6$, the mean is 0.02370.

To compute the standard deviation, we use the formula

$$\sigma(R) = w_C^2 V(R_C) + w_{HP}^2 V(R_{HP}) + w_M^2 V(R_M) + \\ 2(w_C w_{HP} Cov(R_C, R_{HP}) + w_C w_M Cov(R_C, R_M) + w_{HP} w_M Cov(R_{HP}, R_M)),$$

where $V(R) = \sigma(R)^2$ and $Cov(R_1, R_2) = \rho(R_1, R_2)\sigma(R_1)\sigma(R_2)$. The standard deviation of the equally weighted portfolio is 0.05335 and of the 3/2/1 portfolio is 0.05870.

(c) The standard deviation and the weights are in table 1, which is attached. We used Solver to compute portfolio weights for means 3% and 1.2%. As we saw in class, if we know the portfolio weights for any two means, we can deduce the weights for all other means. To find the portfolio weights for mean $x\%$, we write x as a linear combination of 3 and 1.2, i.e. as

$$x = \frac{x-1.2}{3-1.2} \times 3 + \frac{3-x}{3-1.2} \times 1.2.$$

We then multiply the portfolio weights for mean 3% by $(x-1.2)/(3-1.2)$ and the portfolio weights for mean 1.2% by $(3-x)/(3-1.2)$. This trick does not work for the standard deviation. To compute the standard deviation, we simply program the standard deviation as a function of the weights.

(d) The portfolio frontier is the hyperbola in figure 1. Figure 1 shows also the three companies, the portfolio frontier with the riskless asset, needed in part (e), and the tangent portfolio. Notice that McDonald's is "almost" on the frontier. We could have expected this from table 1. If the mean is 1.7%, the weight on McDonald's is 1.002, i.e. very close to 1, while the weights on the other two assets are very close to 0.

When the desired mean is 2.6% or above, McDonald's, the low mean stock, is sold short. As the mean decreases, the weight on McDonald's becomes positive, and the weights on Caterpillar and Hewlett-Packard decrease. The weight on Caterpillar becomes negative if the mean is 1.7% or below, while the weight on Hewlett-Packard becomes negative if the mean is 1.6% or below. The weight on Caterpillar decreases faster that the weight on HP, because the correlation between Caterpillar and McDonald's is higher than the correlation between HP and McDonald's. As the weight on McDonald's increases, it is more important to reduce the weight on Caterpillar than the weight on HP.

(e) The portfolio frontier is in figure 1. To determine the portfolio frontier, we need to use Solver only once. We fix an arbitrary mean, higher than the return on the riskless asset, and find the minimum standard deviation, assuming that we can invest in the risky assets and in the riskless asset. If the mean is, say, 3%, the standard deviation is 0.0766. We then draw the line that goes through the point (0.0766,0.03) and the point (0,0.006).

There are two ways to find the tangent portfolio. First, we can eyeball it from the figure. A more precise way is to eyeball the mean from the figure, and then use Solver. We find the minimum variance for means close to the eyeballed mean, and choose the mean that makes the sum of the weights on the risky assets closest to 1. A close approximation to the mean of the tangent portfolio is 0.02226, and a close approximation to the tangent portfolio is

$$w_C^* = 0.3775, \quad w_{HP}^* = 0.2486, \quad w_M^* = 0.3739.$$

The buck to the bang ratio for Caterpillar is

$$\frac{E(R_C) - R_f}{2Cov(R_C, w_C^* R_C + w_{HP}^* R_{HP} + w_M^* R_M)}.$$

Noting that

$$Cov(R_C, w_C^* R_C + w_{HP}^* R_{HP} + w_M^* R_M) = \\ w_C^* V(R_C) + w_{HP}^* Cov(R_C, R_{HP}) + w_M^* Cov(R_C, R_M),$$

we find that the buck to the bang ratio is 3.0215. The ratio for HP is 3.0215 and for McDonald's is 3.0214, i.e. very close.

6. (a) We regress the excess returns on the stocks to the excess return on the S&P500. The Excel output is attached. Caterpillar's beta is estimated at 1.16. The standard deviation of the estimate is 0.35. HP's beta is estimated at 1.74 with a standard deviation of 0.41, and McDonald's beta is estimated at 0.91 with a standard deviation of 0.26.

   (b) The results are in the following table

	alpha	st. dev. of alpha	st. dev. of id. risk
Caterpillar	0.011	0.009	0.068
HP	0.007	0.011	0.077
McDonald's	0.005	0.007	0.049

   An interesting fact is that betas and idiosyncratic risk go together. HP has both the highest beta and the highest standard deviation of idiosyncratic risk. McDonald's has both the lowest beta and the lowest standard deviation of idiosyncratic risk.

   (c) According to the CAPM, HP should have the highest expected return (since it has the highest beta) and McDonald's the lowest. The sample average for Caterpillar is 0.0245, for HP is 0.0260, and for McDonald's is 0.0167. Therefore these predictions of the CAPM are supported by the data.