

First Half Summary

Present Values

- **Basic Idea:** We should discount future cash flows. The appropriate discount rate is the opportunity cost of capital.
- **Net Present Value:** The net present value of a stream of yearly cash flows is

$$NPV = C_0 + \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)^2} + \cdots + \frac{C_n}{(1+r_n)^n},$$

where r_n is the n year discount rate.

- **Monthly Rate:** The monthly rate, x , is

$$x = (1 + EAR)^{\frac{1}{12}} - 1,$$

where EAR is the effective annual rate. The EAR is

$$EAR = (1 + x)^{12} - 1.$$

- **APR:** Rates are quoted as annual percentage rates (APR's) and not as EAR's. If the APR is monthly compounded, the monthly rate is

$$x = \frac{APR}{12}.$$

- **Perpetuities:** The present value of a perpetuity is

$$PV = \frac{C_1}{r},$$

where C_1 is the cash flow and r the discount rate. This formula assumes that the first payment is after one period.

- **Annuities:** The present value of an annuity is

$$PV = C_1 \left(\frac{1}{r} - \frac{1}{r(1+r)^t} \right),$$

where C_1 is the cash flow, r the discount rate, and t the number of periods. This formula assumes that the first payment is after one period.

Capital Budgeting Under Certainty

- **The NPV Rule:** We should accept a project if its NPV is positive. If there are many mutually exclusive projects with positive NPV, we should accept the project with highest NPV. The NPV rule is the right rule to use.
- **The Payback Rule:** We should accept a project if its payback period is below a given cutoff. If there are many mutually exclusive projects below the cutoff, we should accept the project with shortest payback period. There are two problems with the payback rule. First, it does not take into account cash flows after the cutoff. Second, it does not discount cash flows. Discounted payback fixes the second problem but not the first.
- **The Internal Rate of Return (IRR):** The IRR of a project is the discount rate in the NPV calculation that makes the NPV equal to zero.
- **The IRR Rule:** We should accept a project if its IRR is above the discount rate. If there are many mutually exclusive projects above the discount rate, we should accept the project with highest IRR.
- **Problems with IRR:** IRR is a rate specific to a project and not the relevant discount rate. As a result, it may be misleading when comparing projects of different life or different scale. A project may have multiple IRR's.

Treasury Securities

- **Bond Jargon:** The par value is the face value of the bond. The coupon rate is the bond's interest payment. Coupons are typically paid semiannually. The time to maturity is the length of time that the bond is outstanding. The yield to maturity (YTM) is the IRR of the bond.
- **Treasury Bills and Strips:** Treasury bills and strips are zero coupon bonds. Prices of Treasury bills are quoted in terms of bank discounts

$$\text{bank discount} = \frac{360}{n}(100 - \text{Price}),$$

where n is the number of days to maturity.

- **Forward Rates:** The forward rate that holds from year n to year $m + n$ is given by

$$(1 + {}_n f_m)^m = \frac{(1 + r_{m+n})^{m+n}}{(1 + r_n)^n},$$

where r_n is the n -year spot rate. The forward rate is the rate at which today we can arrange to borrow or lend from year n to year $m + n$.

- **Coupon Bonds:** The price of a coupon bond, P , and the YTM, y , are related by

$$\text{Price} = \sum_{i=0}^{n-1} \frac{C}{\left(1 + \frac{y}{2}\right)^{j + \frac{z}{x}}} + \frac{100}{\left(1 + \frac{y}{2}\right)^{n-1 + \frac{z}{x}}}$$

where n = number of coupon dates until maturity, z = number of days until next coupon date, x = total number of days between last and next coupon dates, C = coupon payment as % of par. Treasury coupon bonds are Treasury notes and Treasury bonds.

- **Bootstrapping:** We can obtain the YTM of coupon bonds by the spot rates, i.e. the YTM of strips. Reversely, we can obtain the spot rates by the YTM of coupon bonds, using bootstrapping.

Interest Rate Risk, Duration, and Convexity

- **Interest Rate Risk:** When bond yields go up, prices go down. We are interested in $\Delta P/\Delta y$, where ΔP is the change in price due to the change Δy in yield.
- **Macaulay Duration (D):** Macaulay duration is

$$D = \frac{1}{2} \sum_{j=1}^n w_j j$$

where the weights w_j are

$$w_j = \frac{\text{coupon}}{P(1+u)^j}, \quad j = 1, \dots, n-1,$$
$$w_n = \frac{\text{coupon} + \text{par value}}{P(1+u)^n},$$

j = coupon date, n = number of coupon dates until maturity, $u \equiv \frac{y}{2}$, and y = YTM. The formula assumes that there is no accrued interest, i.e. the next coupon payment is in 6 months. Macaulay duration is the average period of bond payments.

- **Modified Duration (D^*):** Modified duration is

$$D^* = \frac{D}{(1+u)}.$$

Modified duration is related to the sensitivity of the bond price to changes in the YTM through

$$\frac{\partial P}{\partial y} = -D^* P.$$

Using modified duration, we can obtain a first-order approximation of $\Delta P/\Delta y$.

- **Convexity (Γ):** Convexity is

$$\frac{\partial^2 P}{\partial y^2} = \Gamma P.$$

Using modified duration and convexity, we can obtain a second-order approximation of $\Delta P/\Delta y$.

- **Fixed-Income Portfolio Management:** Duration and convexity are very useful in fixed-income portfolio management.

Corporate Bonds and the Term Structure

- **Corporate Bond Jargon:** Corporate bonds have a non-zero probability of default, unlike Treasury bonds. Their YTM is defined as for Treasury bonds and is higher than the YTM of comparable Treasury bonds. Corporate bonds can be secured or unsecured, and senior or subordinated. They generally have a sinking fund, put and call provisions, and covenants. Corporate bonds are rated by independent agencies.
- **The Term Structure of Interest Rates:** The relation between the spot rate and the time to maturity is the term structure of interest rates.
- **Expectations Hypothesis:** The expectations hypothesis states that the term structure reflects expectations of future spot rates. More precisely, the expected spot rate is equal to the forward rate, i.e.

$$E({}_i r_n) = {}_i f_n.$$

- **Liquidity Preference Hypothesis:** The liquidity preference hypothesis states that risk-averse investors prefer short-term investments. The forward rate is greater than the expected spot rate, and the difference is a liquidity premium.

Valuation of Common Stocks

- **Present Value Again:** The price of a stock is

$$P_0 = \frac{D_1}{(1+r_1)} + \frac{D_2}{(1+r_1)(1+r_2)} + \frac{D_3}{(1+r_1)(1+r_2)(1+r_3)} + \dots$$

i.e. it is the present value of expected dividends discounted at investors' expected return. If dividends are expected to grow at a constant rate, g , we get the **dividend growth model** (DGM)

$$P_0 = \frac{D_1}{r-g}.$$

- **Uses of the DGM:** To use the DGM we need to estimate dividend growth, g . We can estimate g using historical or forecasted growth, or ROE and the payout ratio. A first use of the DGM is to get an estimate of the price of the stock. (Since markets are efficient, this estimate is of limited use.) We can also use the DGM to estimate the market capitalization rate,

$$\hat{r} = \frac{D_1}{P_0} + g.$$

- **PVGO and P/E ratio:** The present value of growth opportunities (PVGO) is

$$\text{PVGO} = \text{Price} - \text{no-growth price} = \text{Price} - \frac{\text{EPS}_1}{\hat{r}}.$$

It is a measure of the growth opportunities of a company. The price-earnings ratio (P/E ratio) is

$$\text{P/E} = \frac{P_0}{\text{EPS}_0}.$$

Measuring Risk

- **Expected Return and Sample Mean:** The expected return or mean return is

$$E(R) = \sum_j R_j p_j,$$

where j denotes a particular scenario, R_j the return under that scenario, and p_j the probability of the scenario. The sample mean is

$$\bar{R} = \frac{\sum_i R_i}{N}.$$

It is an estimate of the true mean.

- **Variance and Standard Deviation:** The variance and standard deviation are measures of dispersion. The variance is

$$V(R) = \sum_j (R_j - E(R))^2 p_j,$$

and the standard deviation is

$$\sigma(R) = \sqrt{V(R)}.$$

The sample standard deviation is

$$s = \sqrt{\frac{\sum_i (R_i - \bar{R})^2}{N - 1}}.$$

- **Covariance and Correlation:** The covariance and correlation are measures of association. The covariance is

$$Cov(R_1, R_2) = \sum_j (R_{1,j} - E(R_1))(R_{2,j} - E(R_2))p_j,$$

and the correlation is

$$\rho(R_1, R_2) = \frac{Cov(R_1, R_2)}{\sigma(R_1)\sigma(R_2)}.$$

The correlation is always between -1 and 1. The sample covariance is

$$Cov(R_1, R_2) = \frac{\sum_i (R_{1,i} - \bar{R}_1)(R_{2,i} - \bar{R}_2)}{N - 1},$$

and the sample correlation is

$$\rho(R_1, R_2) = \frac{Cov(R_1, R_2)}{s(R_1)s(R_2)}.$$

- **Portfolio Returns:** The return on a portfolio with n assets is

$$R = \sum_{k=1}^n w_k R_k,$$

where R_k is the return on the k 'th asset, and w_k is the asset's weight. The expected return is

$$E(R) = \sum_{k=1}^n w_k E(R_k),$$

and the variance is

$$V(R) = \sum_{k=1}^n w_k^2 V(R_k) + 2 \sum_{k > \ell} w_k w_\ell Cov(R_k, R_\ell).$$

Portfolio Theory

- **Portfolio Frontier:** A portfolio is a frontier portfolio if it has the minimum variance among portfolios that have the same expected return. The portfolio frontier is the set of all frontier portfolios. The part of the portfolio frontier that lies above the minimum-variance portfolio is the efficient frontier.
- **The Benefits and Limits of Diversification:** Diversification reduces the **idiosyncratic risk** of assets. However, it does not affect the **systematic** risk.
- **Shape of the Portfolio Frontier:** With risky assets only, the portfolio frontier is a hyperbola. If there is also a riskless asset, the portfolio frontier is a line.
- **The Tangent Portfolio:** The point where the line is tangent to the hyperbola is called the tangent portfolio. The tangent portfolio is such that the **buck for the bang** ratio

$$\frac{\text{Increase in Expected Return by Investing in Asset } i}{\text{Increase in Variance by Investing in Asset } i} = \frac{E(R_i) - R_f}{2Cov(R_i, R^*)}$$

is independent of the particular risky asset i .

- **The Optimal Portfolio:** An investor's optimal portfolio is a combination of the tangent portfolio and the riskless asset. The weights depend on the investor's risk-aversion.

The Capital Asset Pricing Model (CAPM)

- **The Market Portfolio:** The market portfolio is the portfolio formed by all risky assets. In market equilibrium the tangent portfolio is the market portfolio.
- **The CAPM:** The CAPM is

$$E(R_i) - R_f = \beta_i(E(R_m) - R_f),$$

where

$$\beta_i = \frac{\text{Cov}(R_i, R_m)}{V(R_m)}.$$

The CAPM links the expected excess return on asset i to the asset's beta and to the expected excess return on the market portfolio.

- **Beta:** We can always write the excess return on asset i as

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + \epsilon_i.$$

The first term is a constant, the second term represents systematic risk, and the third term represents idiosyncratic risk. Beta measures systematic risk. It tells us how much the return on asset i increases on average, if the return on the market portfolio increases by 1%.

- **The CAPM's Basic Idea:** The CAPM states that the expected return of asset i depends on the asset's risk only through the beta (and not through the variance). In other words, it is systematic risk and not idiosyncratic risk that is priced in the market.
- **The CAPM's Intuition:** The CAPM implies that an asset with zero beta has the same return as the riskless asset. The intuition is that this asset's risk is only idiosyncratic and can be diversified away. The CAPM also implies that an asset with negative beta has lower return than the riskless asset. The intuition is that this asset reduces portfolio risk.
- **The Capital Market Line (CML) and the Security Market Line (SML):** The CML is the line that goes through the riskless asset and the market portfolio in the standard deviation/mean space. Only frontier portfolios are on this line. The SML is line that goes through the riskless asset and the market portfolio in the beta/mean space. According to the CAPM, all assets are on this line.
- **Estimating Beta:** To estimate beta we use linear regression. Regression gives us also an idea of how precise is our estimate of beta, and what is the standard deviation of idiosyncratic risk.