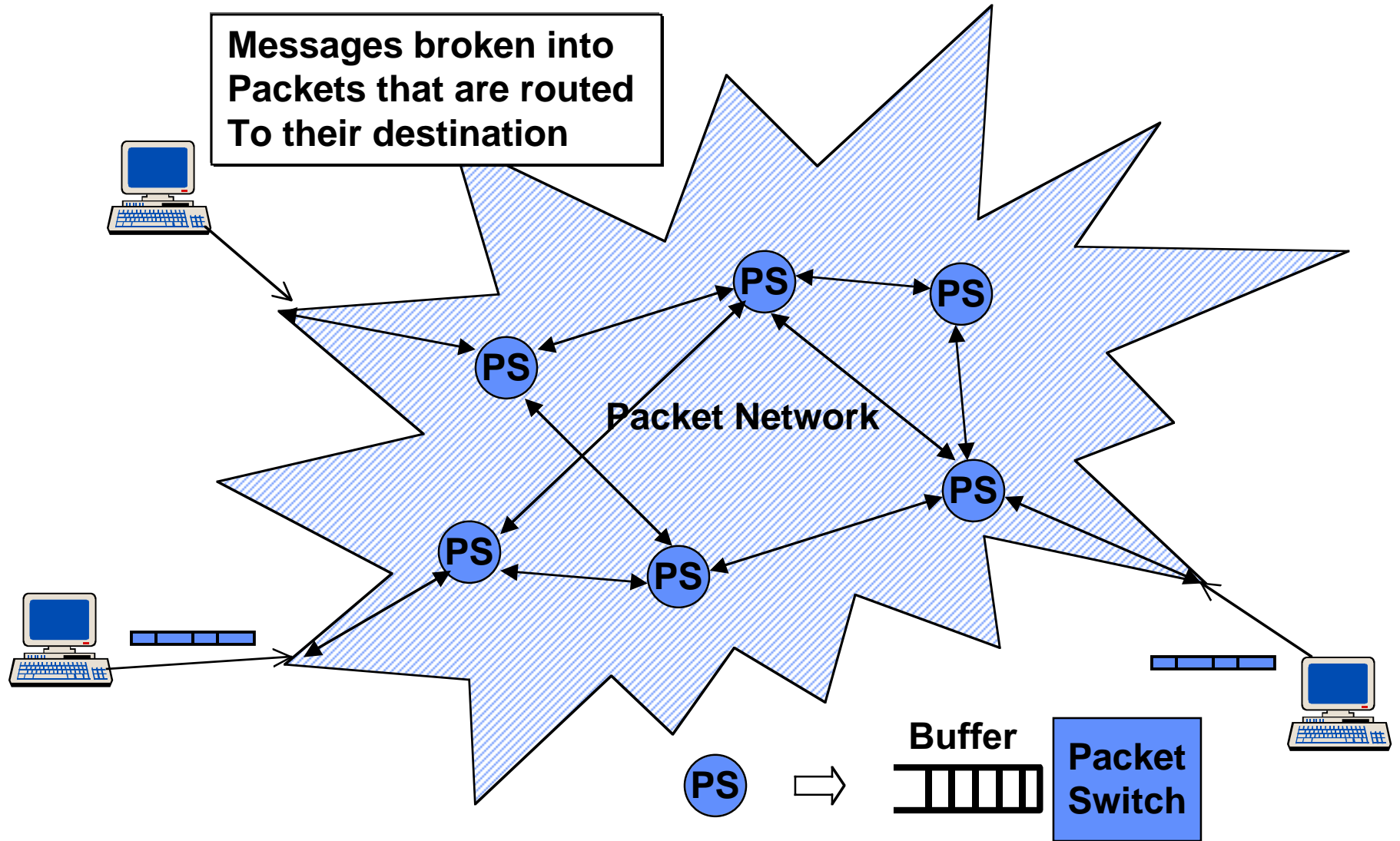

Routing in Data Networks

Eytan Modiano

Packet Switched Networks



Routing

- **Must choose routes for various origin destination pairs (O/D pairs) or for various sessions**
 - **Datagram routing: route chosen on a packet by packet basis**

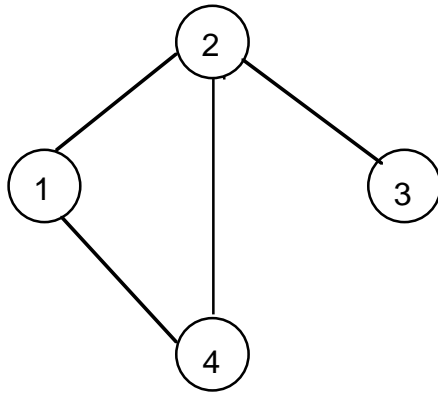
Using datagram routing is an easy way to split paths
 - **Virtual circuit routing: route chosen a session by session basis**
 - **Static routing: route chosen in a prearranged way based on O/D pairs**

Broadcast Routing

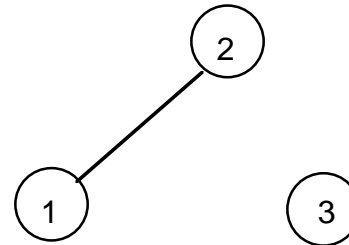
- **Route a packet from a source to all nodes in the network**
- **Possible solutions:**
 - **Flooding: Each node sends packet on all outgoing links**
Discard packets received a second time
 - **Spanning Tree Routing: Send packet along a tree that includes all of the nodes in the network**

Graphs

- A graph $G = (N,A)$ is a finite nonempty set of nodes and a set of node pairs A called arcs (or links or edges)



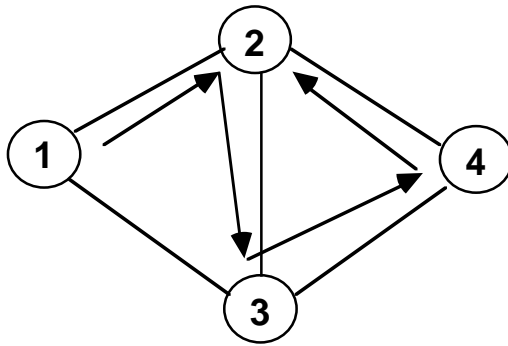
$$N = \{1,2,3,4\}$$
$$A = \{(1,2),(2,3),(1,4),(2,4)\}$$



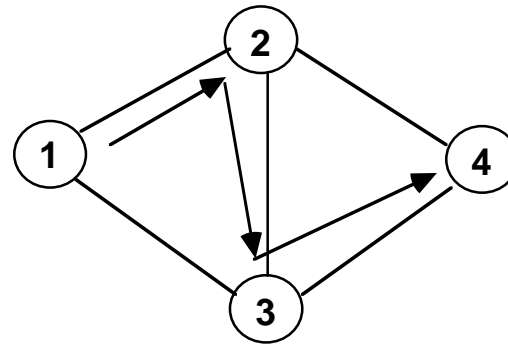
$$N = \{1,2,3\}$$
$$A = \{(1,2)\}$$

Walks and paths

- A walk is a sequence of nodes (n_1, n_2, \dots, n_k) in which each adjacent node pair is an arc.
- A path is a walk with no repeated nodes.



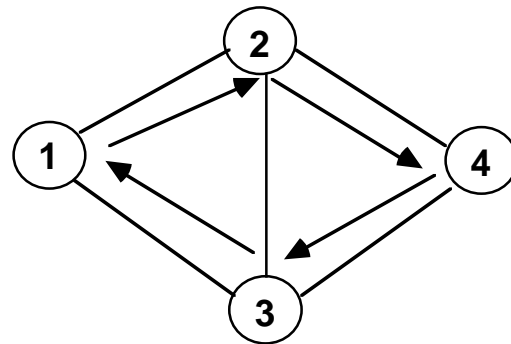
Walk (1,2,3,4,2)



Path (1,2,3,4)

Cycles

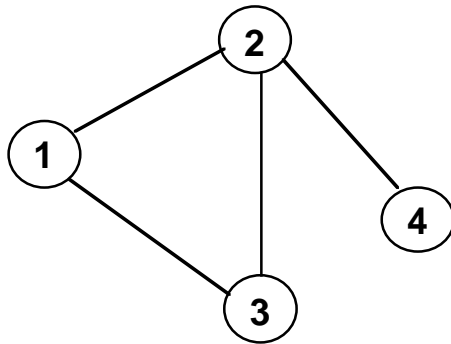
- A cycle is a walk (n_1, n_2, \dots, n_k) with $n_1 = n_k$, $k > 3$, and with no repeated nodes except $n_1 = n_k$



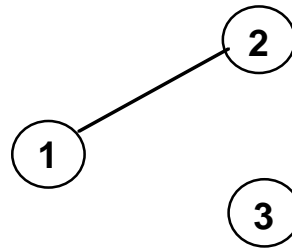
Cycle (1,2,4,3,1)

Connected graph

- A graph is connected if a path exists between each pair of nodes.



Connected

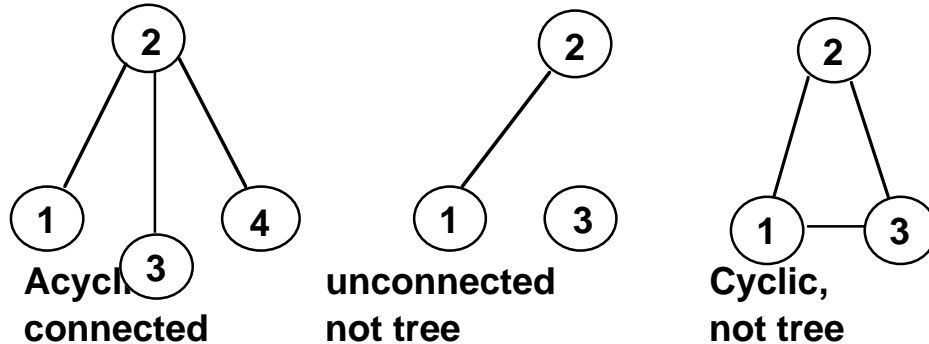


Unconnected

- An unconnected graph can be separated into two or more connected components.

Acyclic graphs and trees

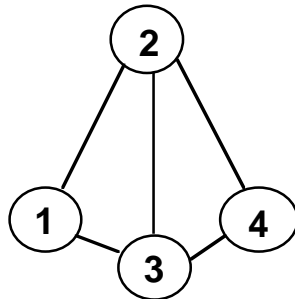
- An acyclic graph is a graph with no cycles.
- A tree is an acyclic connected graph.



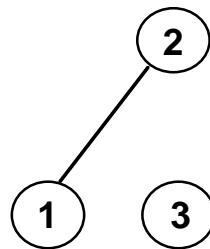
- The number of arcs in a tree is always one less than the number of nodes
 - Proof: start with arbitrary node and each time you add an arc you add a node \Rightarrow N nodes and $N-1$ links. If you add an arc without adding a node, the arc must go to a node already in the tree and hence form a cycle

Subgraphs

- $G' = (N', A')$ is a subgraph of $G = (N, A)$ if
 - 1) G' is a graph
 - 2) N' is a subset of N
 - 3) A' is a subset of A
- One obtains a subgraph by deleting nodes and arcs from a graph
 - Note: arcs adjacent to a deleted node must also be deleted



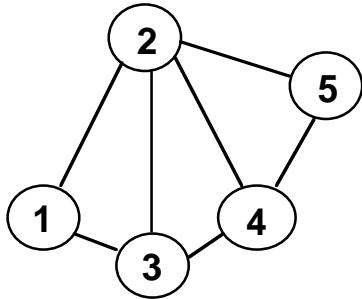
– Graph G



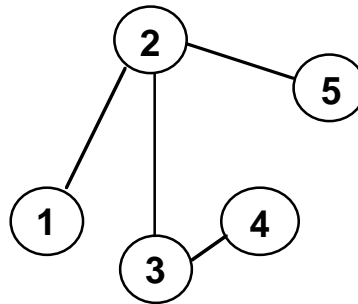
Subgraph G' of G

Spanning trees

- $T = (N', A')$ is a spanning tree of $G = (N, A)$ if
 - T is a subgraph of G with $N' = N$ and T is a tree



Graph G



Spanning tree of G

Spanning trees

- **Spanning trees are useful for disseminating and collecting control information in networks; they are sometimes useful for routing**
- **To disseminate data from Node n:**
 - Node n broadcasts data on all adjacent tree arcs
 - Other nodes relay data on other adjacent tree arcs
- **To collect data at node n:**
 - All leaves of tree (other than n) send data
 - Other nodes (other than n) wait to receive data on all but one adjacent arc, and then send received plus local data on remaining arc

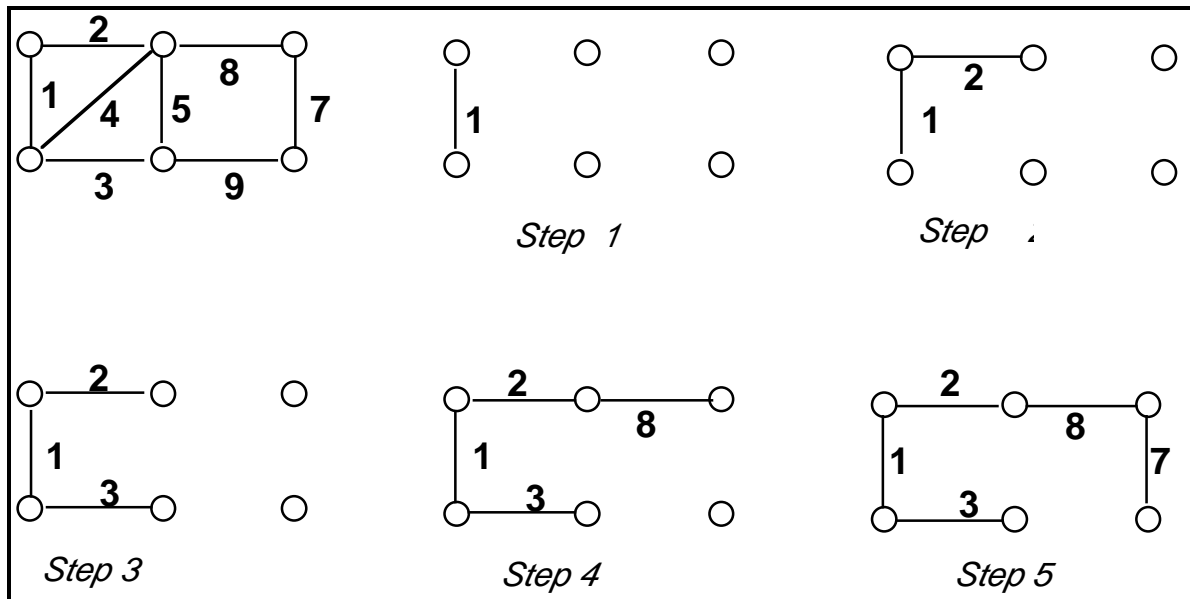
General construction of a spanning tree

- **Algorithm to construct a spanning tree for a connected graph $G = (N,A)$:**
 - 1) Select any node n in N ; $N' = \{n\}$; $A' = \{ \}$
 - 2) If $N' = N$, then stop ($T=(N',A')$ is a spanning tree)
 - 3) Choose $(i,j) \in A$, $i \in N'$, $j \notin N'$
 $N' := N' \cup \{j\}$; $A' := A' \cup \{(i,j)\}$; go to step 2
- **Connectedness of G assures that an arc can be chosen in step 3 as long as $N' \neq N$**
- **Is spanning tree unique?**
- **What makes for a good spanning tree?**

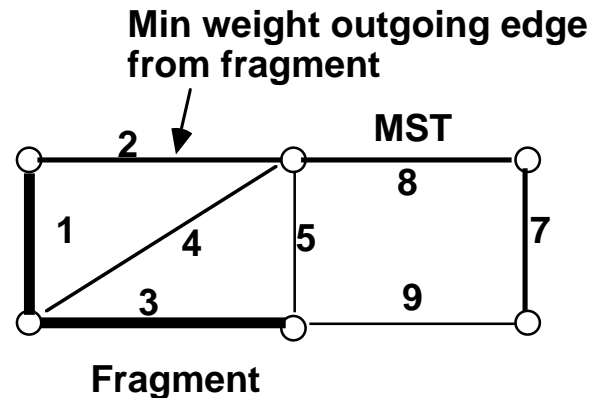
Minimum Weight Spanning Tree (MST)

- **Generic MST algorithm steps:**
 - Given a collection of subtrees of an MST (called fragments) add a minimum weight outgoing edge to some fragment
- **Prim-Dijkstra: Start with an arbitrary single node as a fragment**
 - Add minimum weight outgoing edge
- **Kruskal: Start with each node as a fragment;**
 - Add the minimum weight outgoing edge, minimized over all fragments

Prim-Dijkstra Algorithm



Kruskal's Algorithm Example



- Suppose the arcs of weight 1 and 3 are a fragment
 - Consider any spanning tree using those arcs and the arc of weight 4, say, which is an outgoing arc from the fragment.
 - Suppose that spanning tree does not use the arc of weight 2.
 - Removing the arc of weight 4 and adding the arc of weight 2 yields another tree of smaller weight.
 - Thus an outgoing arc of min weight from fragment must be in MST.