Problem Solving as State Space Search

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16.070

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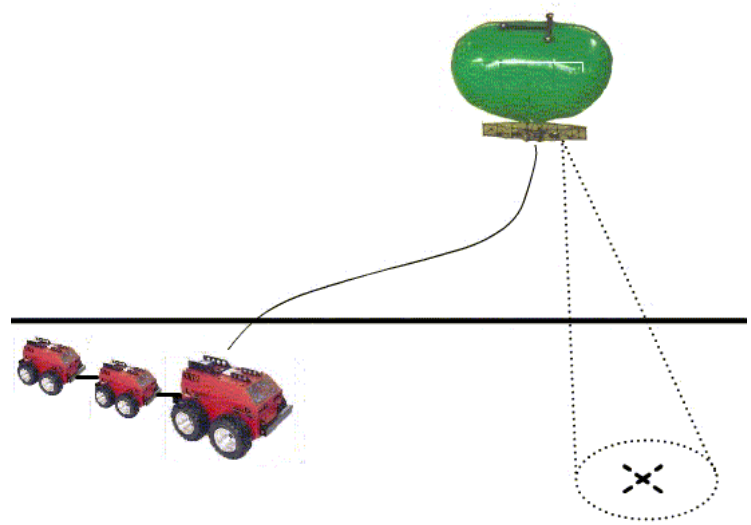
Slides adapted from:
6.034 Tomas Lozano Perez,
Russell and Norvig AIMA


Self-Diagnosing Explorers
courtesy of JPL

In Space The Exception is the Rule

- Quintuple fault occurs (three shorts, tank-line and pressure jacket burst, panel flies off).
- Power limitations too severe to perform new mission..
- Novel reconfiguration identified, exploiting LEM batteries for power.
- Swaggert & Lovell work on Apollo 13 emergency rig lithium hydroxide unit.

APOLLO 13



Complex missions must carefully:

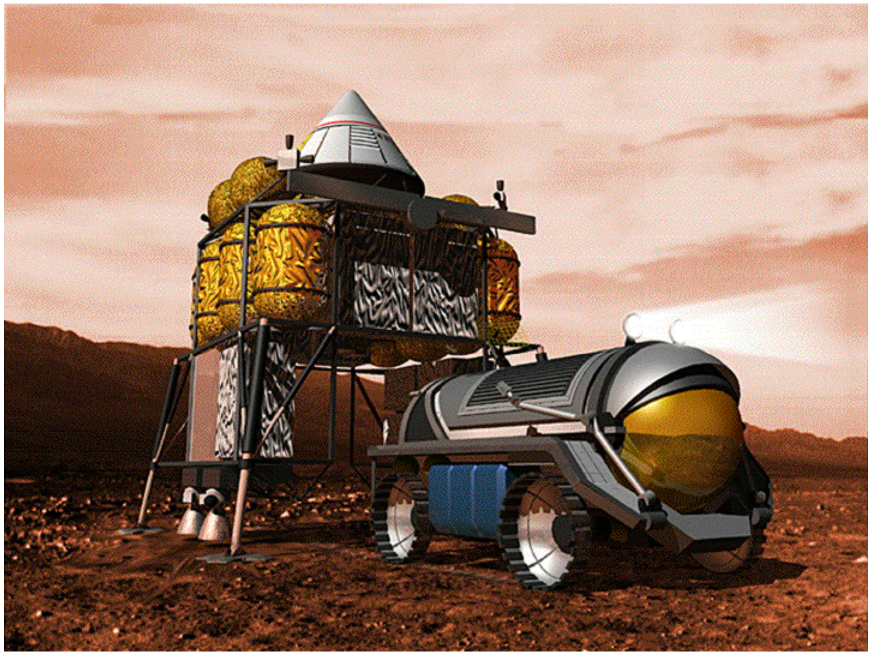
- Plan complex sequences of actions
- Schedule tight resources
- Monitor and diagnose behavior
- Repair or reconfigure hardware.

⇨ Most Autonomy problems, search through a space of options.

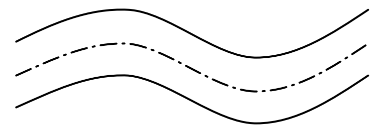
⇨ We formulate as state space search.

Outline

- Problem encoding as state space search
- Graphs and search trees
- Depth and breadth-first search



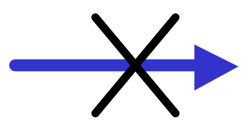
Astronaut
Goose
Grain
Fox

Rover 

- Astronaut + 1 item allowed in the rover.
- Goose alone eats Grain
- Fox alone eats Goose

Can the astronaut get its produce safely across the Martian canal?

Early AI: What are the universal problem solving methods?

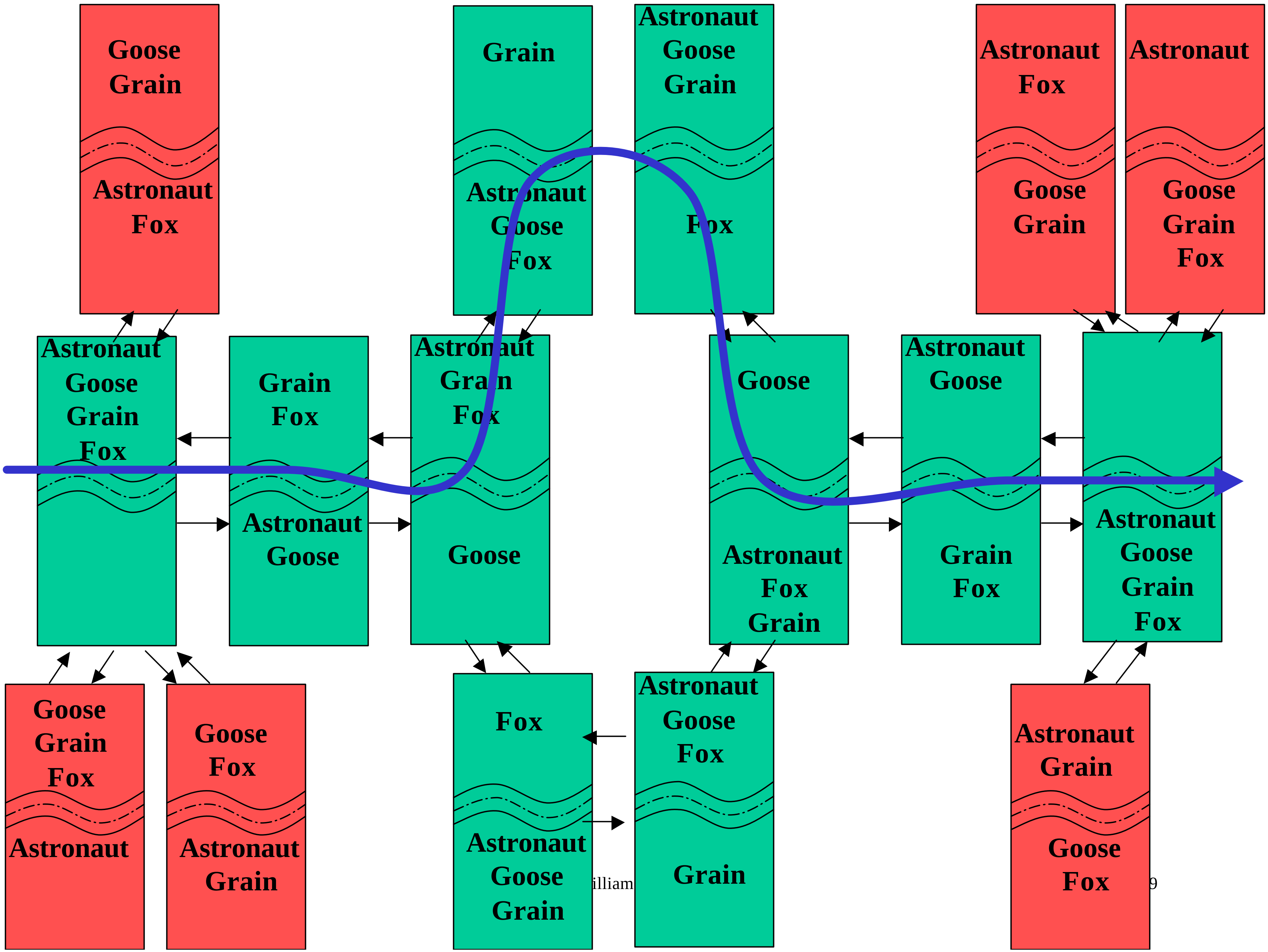
Simple  Trivial

Problem Solving as State Space Search

- Formulate Goal
- Formulate Problem
  - States
  - Operators
- Generate Solution
  - Sequence of states

Problem Solving as State Space Search

- Formulate Goal
  - Astronaut, Fox, Goose & Grain across river
- Formulate Problem
  - States
    - Location of Astronaut, Fox, Goose & Grain at top or bottom river bank
  - Operators
    - Move rover with astronaut & 1 or 0 items to other bank.


Goose
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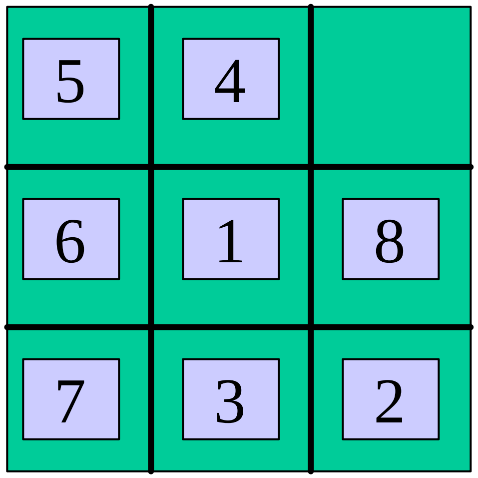
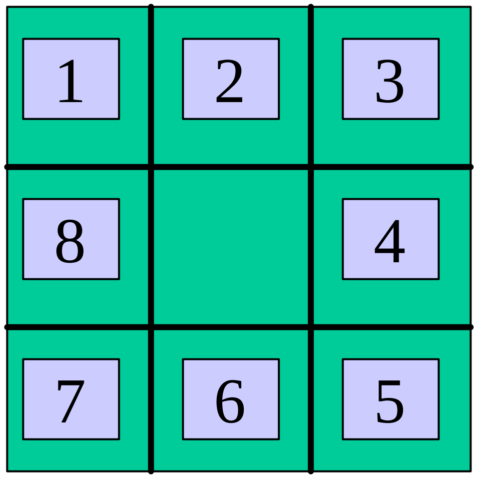
Grain
Goose
Fox

Example: 8-Puzzle



Start



Goal

- States: integer location for each tile AND …
- Operators: move empty square up, down, left, right
- Goal Test: goal state as given

Outline

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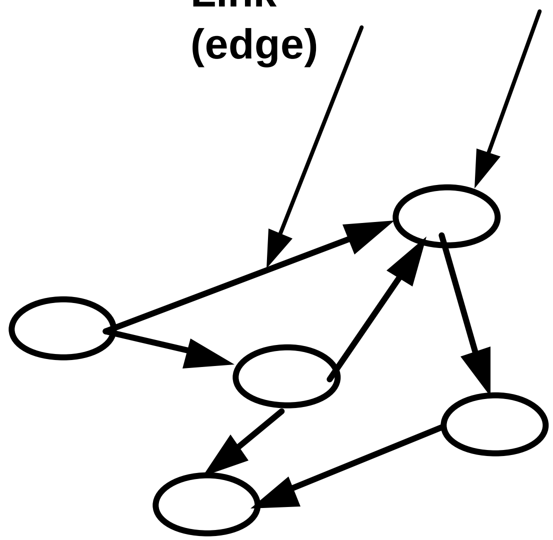
- Depth and breadth-first search

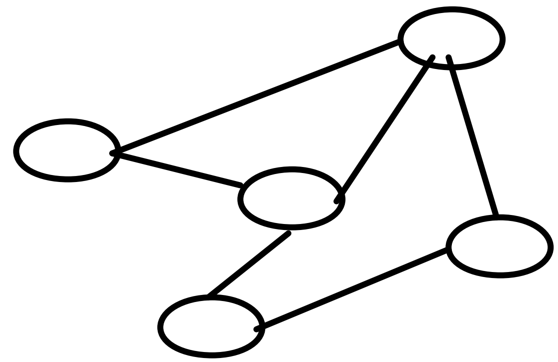
Problem Formulation: A Graph

Operator **State**

Link (edge) **Node (vertex)**

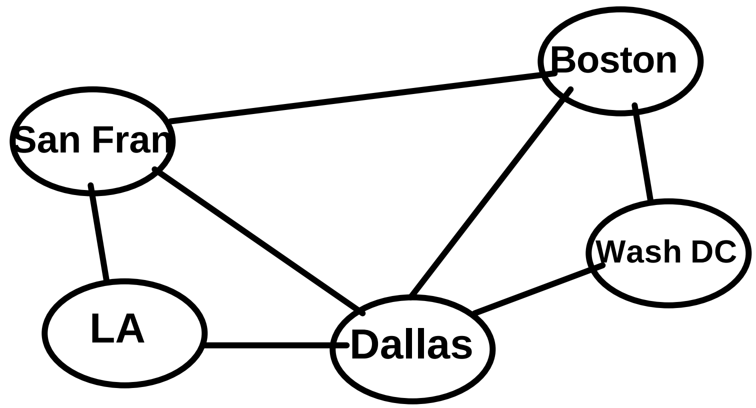


Directed Graph
(one-way streets)



Undirected Graph
(two-way streets)

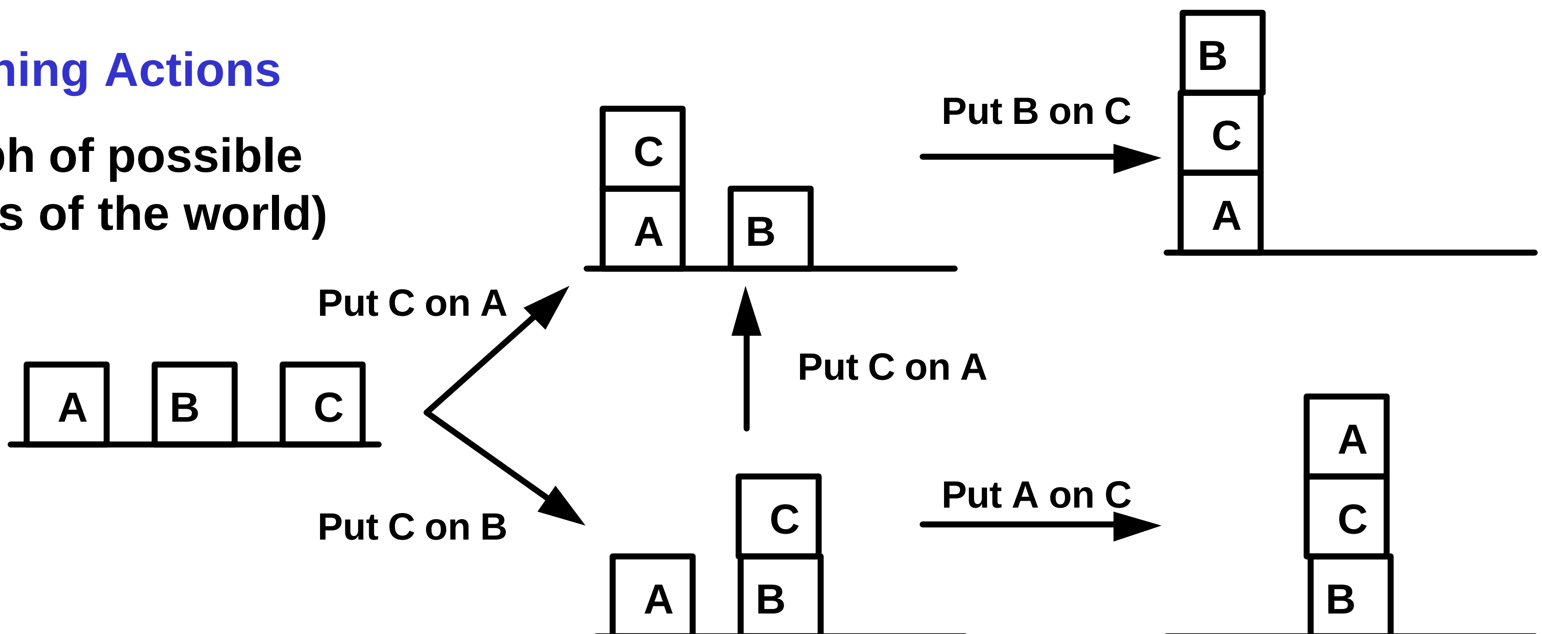
Examples of Graphs



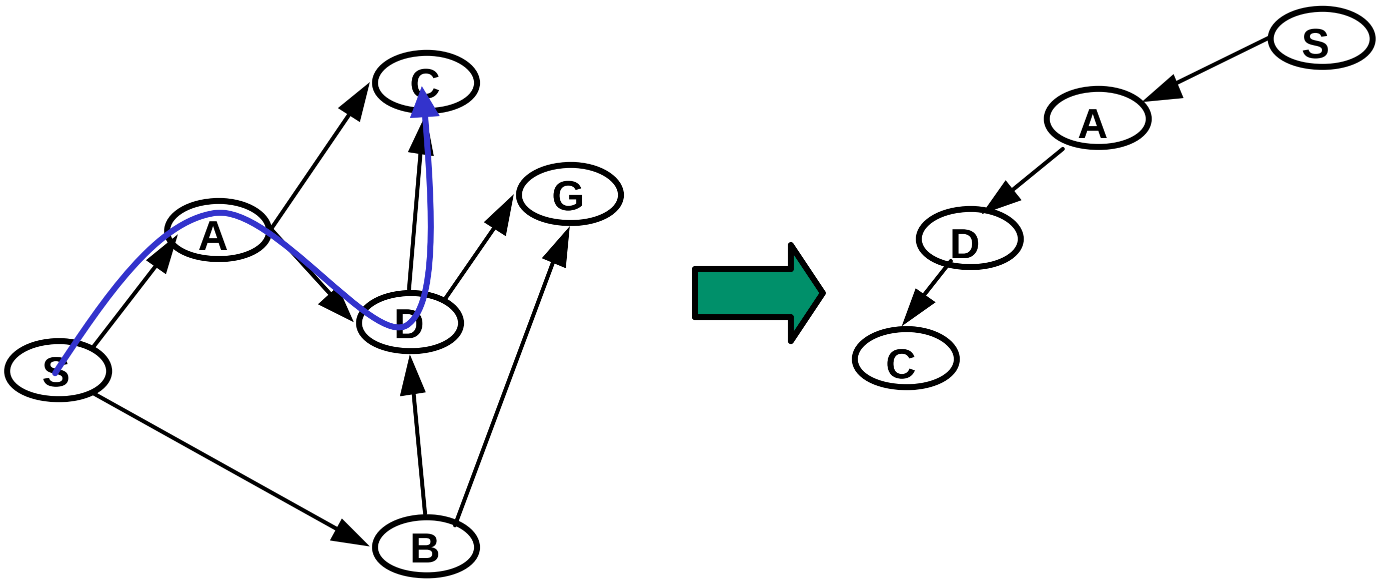
Airline Routes

Planning Actions

(graph of possible states of the world)

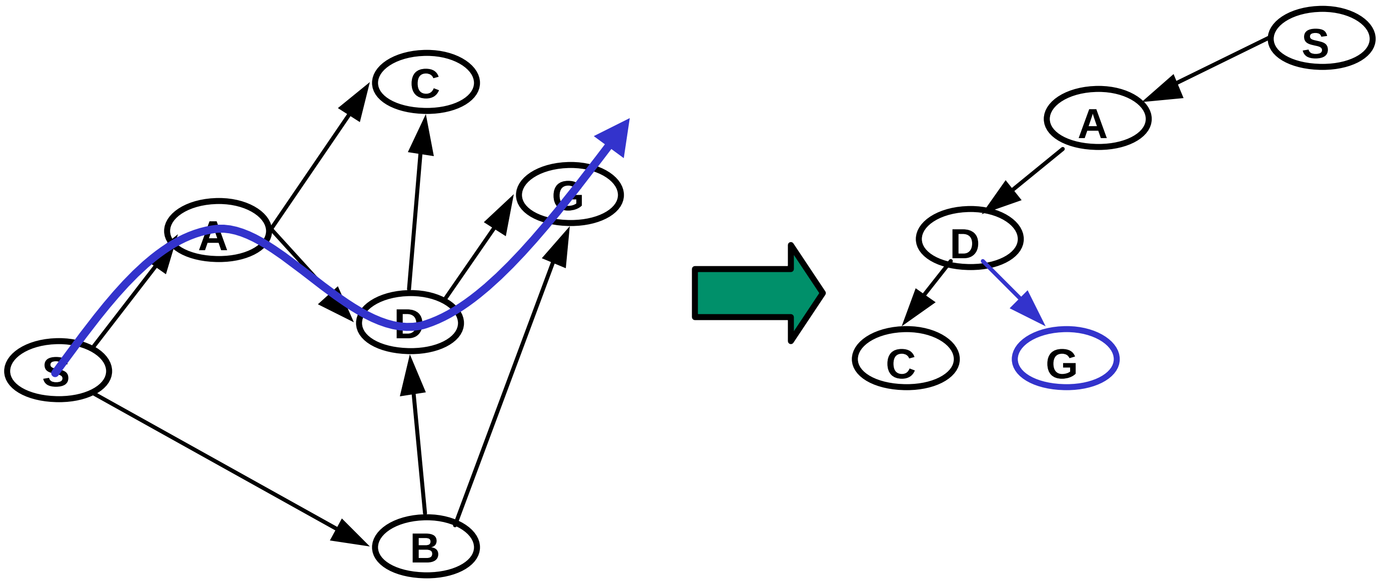


A Solution is a State Sequence: Problem Solving Searches Paths



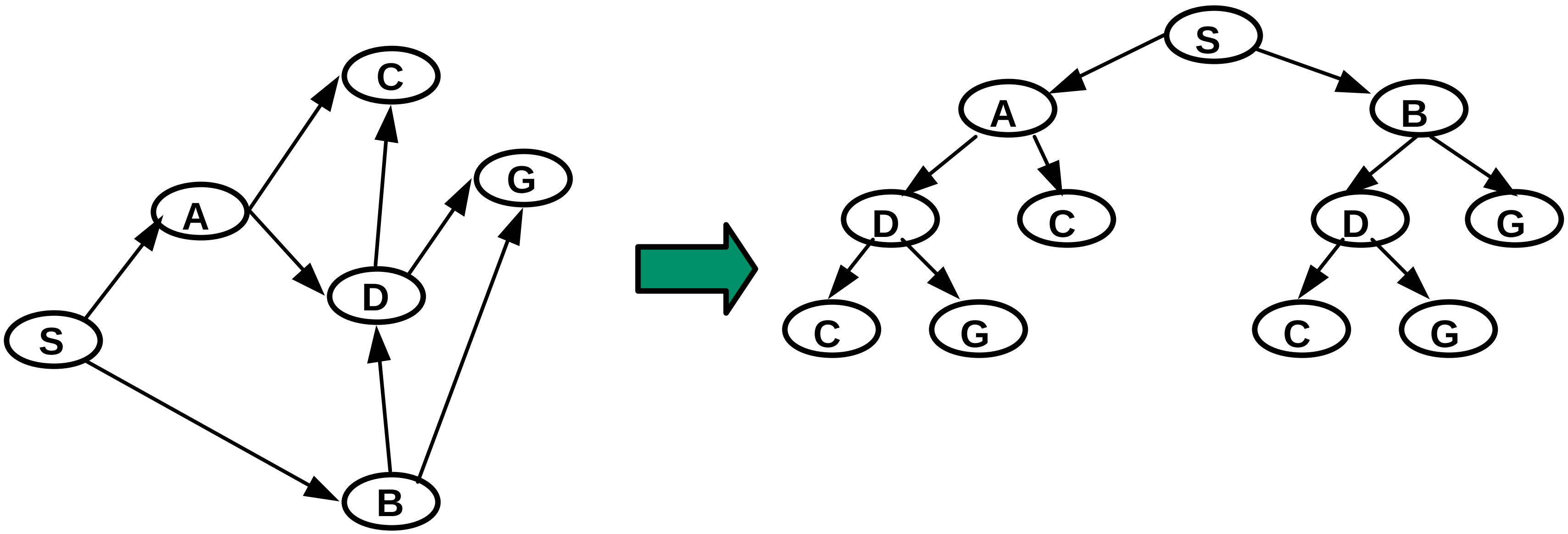
Represent searched paths using a tree.

A Solution is a State Sequence: Problem Solving Searches Paths



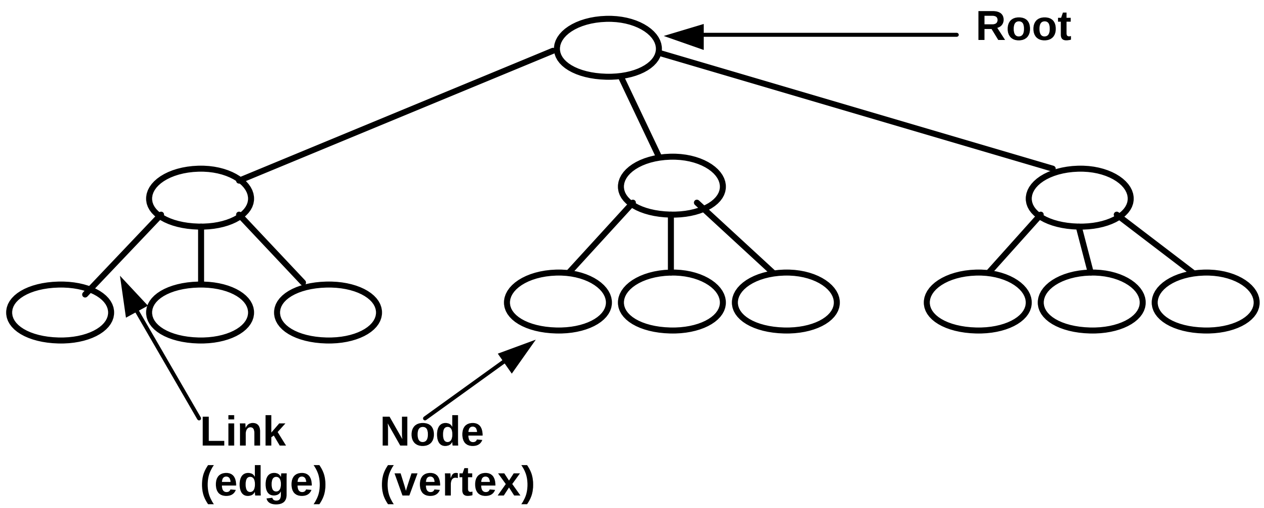
Represent searched paths using a tree.

A Solution is a State Sequence: Problem Solving Searches Paths

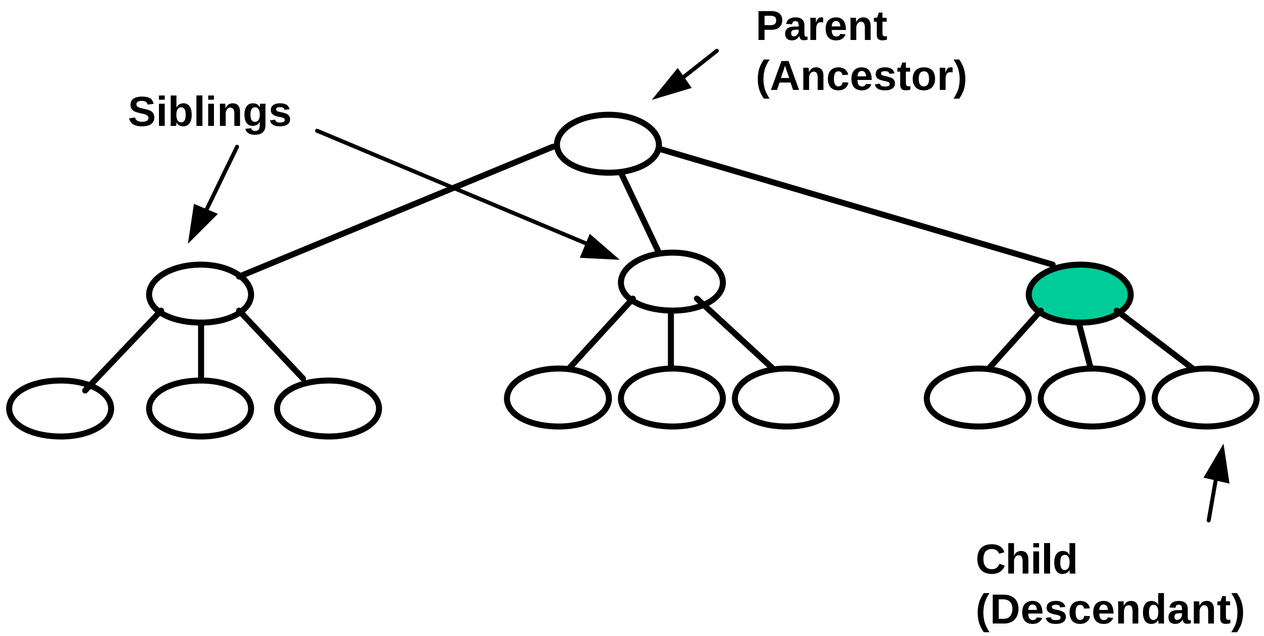


Represent searched paths using a tree.

Search Trees



Search Trees



Outline

- Problem encoding as state space search
- Graphs and search trees
- Depth and breadth-first search
  - Depth first in lecture
  - Breadth first at home

Classes of Search

Blind (uninformed)	**Depth-First** **Breadth-First** **Iterative-Deepening**	Systematic exploration of whole tree until the goal is found.
Heuristic	**Hill-Climbing** **Best-First** **Beam**	Uses heuristic measure of goodness of a node,e.g. estimated distance to. goal.
Optimal	**Branch&Bound** **A***	Uses path “length” measure. Finds “shortest” path. A* also uses heuristic

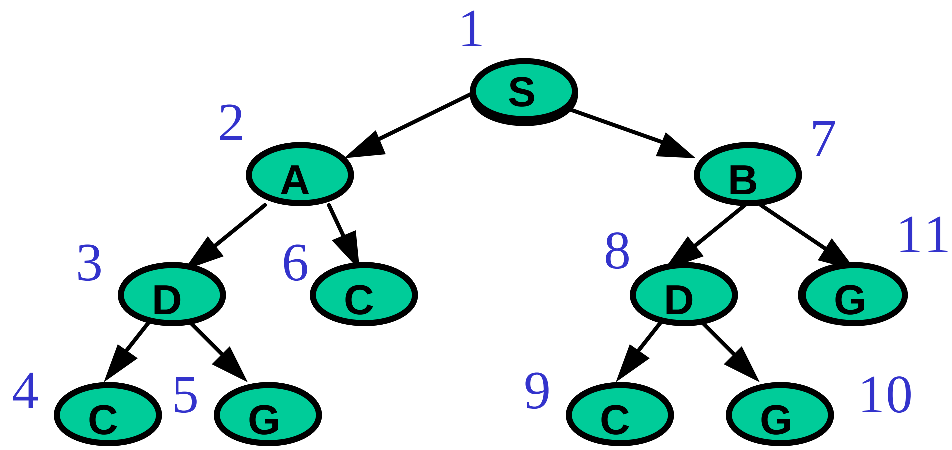
Classes of Search

Blind	**Depth-First**	Systematic exploration of whole tree
(uninformed)	**Breadth-First**	until the goal is found.
	Iterative-Deepening	

Depth First Search (DFS)

Idea:

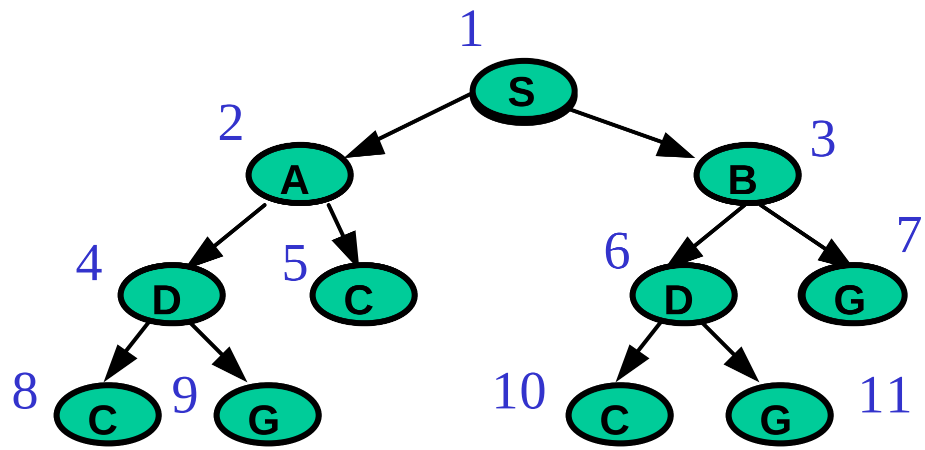
- Explore descendants before siblings
- Explore siblings left to right



Breadth First Search (BFS)

Idea:

- Explore relatives at same level before their children
- Explore relatives left to right



Elements of Algorithm Design

Description:

- stylized pseudo code, sufficient to analyze and implement the algorithm.

Analysis:

- Soundness:
  - is a solution returned by the algorithm guaranteed to be correct?
- Completeness:
  - is the algorithm guaranteed to find a solution when there is one?
- Optimality:
  - is the algorithm guaranteed to find a best solution when there is one?
- Time complexity:
  - how long does it take to find a solution?
- Space complexity:
  - how much memory does it need to perform search?

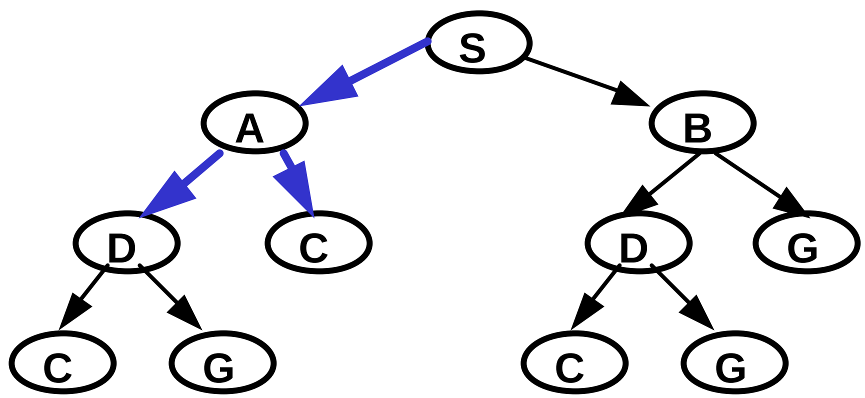
Outline

- Problem encoding as state space search
- Graphs and search trees
- Depth and breadth-first search
  - A generic search algorithm
  - Depth-first search example
  - Handling cycles
  - Breadth-first search example (do at home)

Simple Search Algorithm

How do we maintain the Search State?

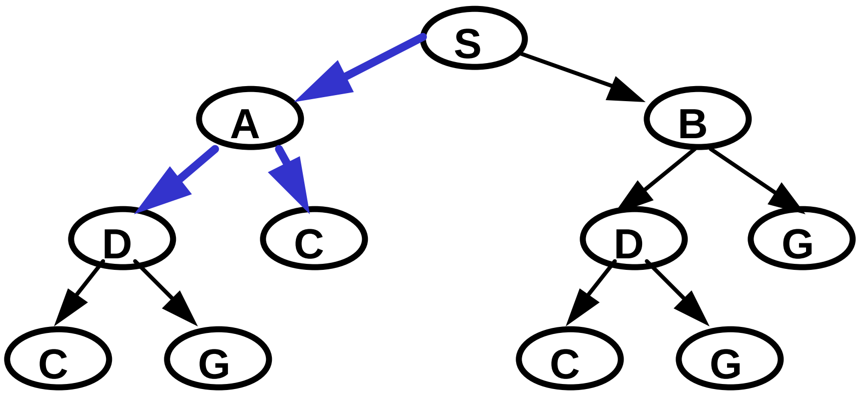
- A set of partial paths explored thus far.
- An ordering on which partial path to expand next (called a queue Q).



- Search repeatedly:
    - Selects next partial path
    - Expands it.
- Terminates when goal found.

Simple Search Algorithm

- Let S denote the start node and G a goal node.
- A partial path is a path from S to some node D,
  - e.g., (D A S)



- The head of a partial path is the most recent node of the path,
  - e.g., D.
- The Q is a list of partial paths,
  - e.g. ((D A S) (C A S) …).

Simple Search Algorithm

Let Q be a list of partial paths,
Let S be the start node and
Let G be the Goal node.

1. **Initialize Q with partial path (S)**
2. **If Q is empty, fail. Else, pick a partial path N from Q**
3. **If head(N) = G, return N** **(goal reached!)**
4. **Else:**
   a) **Remove N from Q**
   b) **Find all children of head(N) and create all the one-step extensions of N to each child.**
   c) **Add all extended paths to Q**
   d) **Go to step 2.**

Outline

- Problem encoding as state space search
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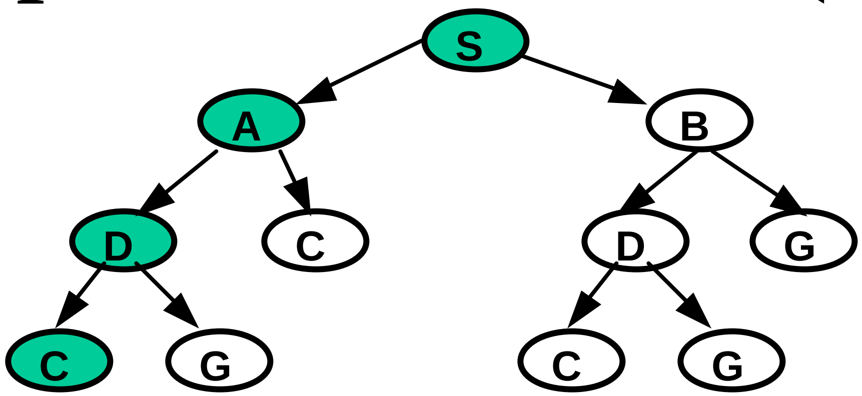
Depth First Search (DFS)



Depth-first:

Add path extensions to **front** of Q

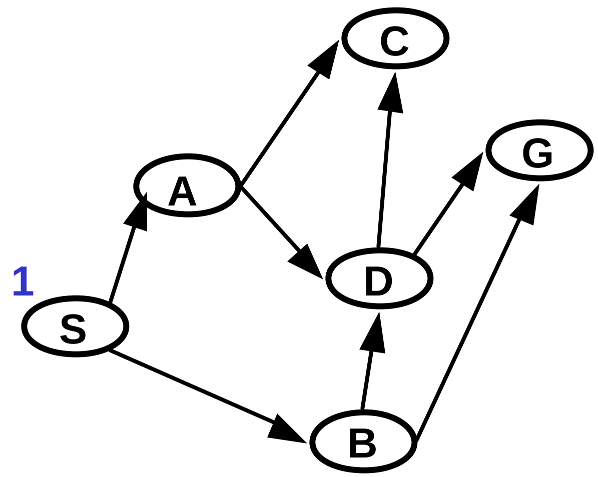
Pick first element of Q

For each search type, where do we place the children on the queue?

Depth-First

Pick first element of Q; Add path extensions to front of Q

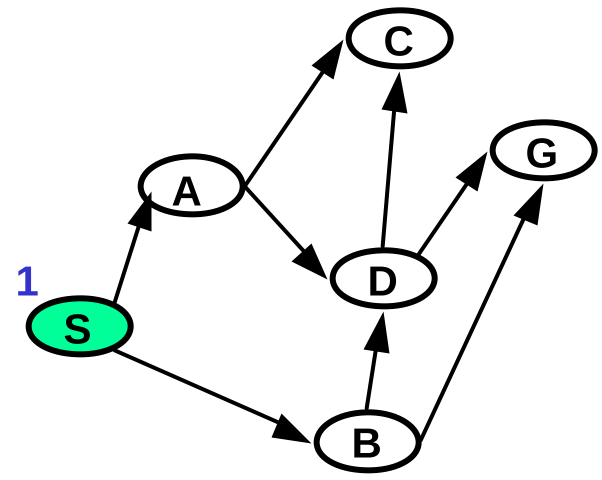
	Q
1	(S)
2	
3	
4	
5	



Depth-First

Pick first element of Q; Add path extensions to front of Q

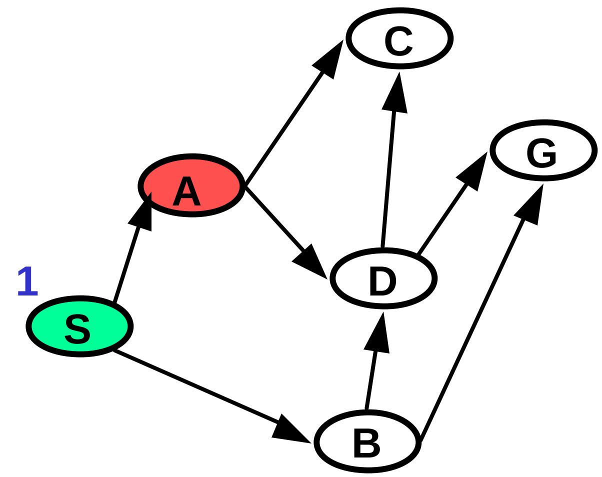
	Q
1	~~(S)~~
2	
3	
4	
5	



Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S)
3	
4	
5	

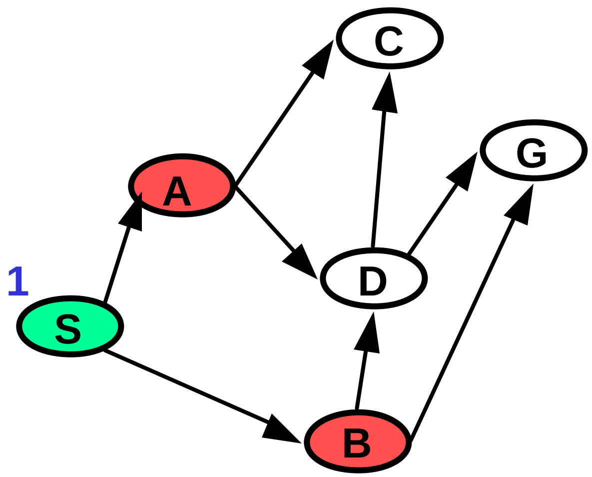


Added paths in blue

Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S) (B S)
3	
4	
5	



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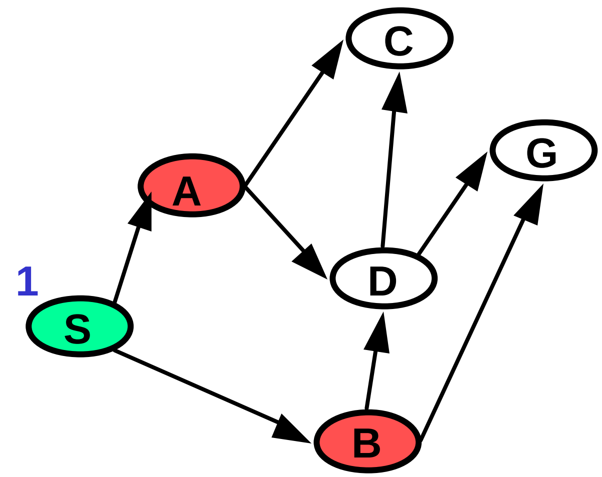

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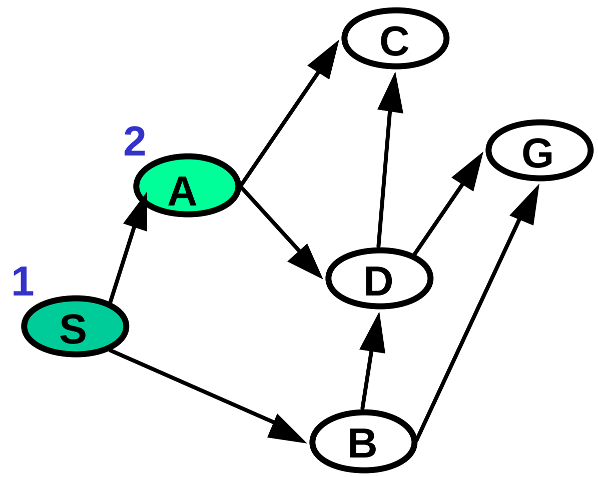

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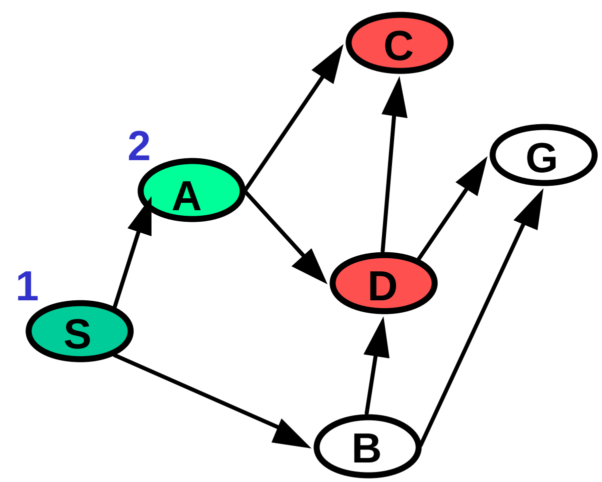
Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	
5	

Added paths in blue



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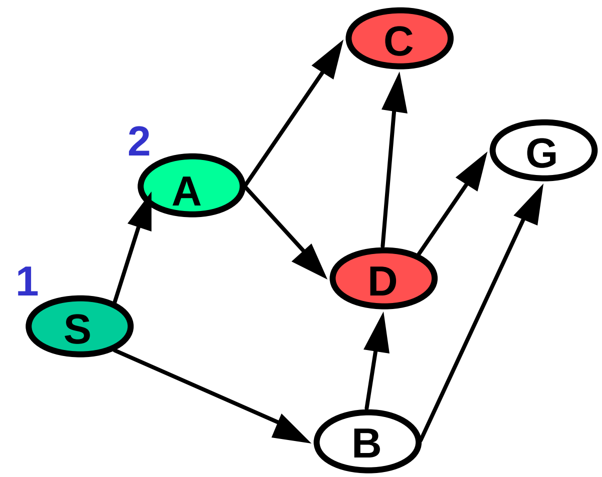

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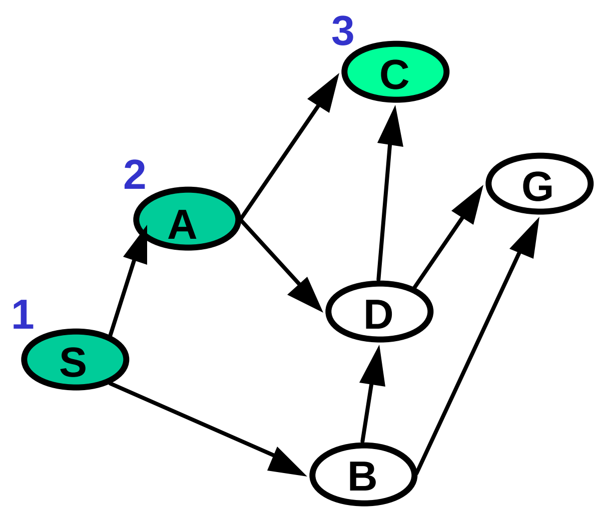

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Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
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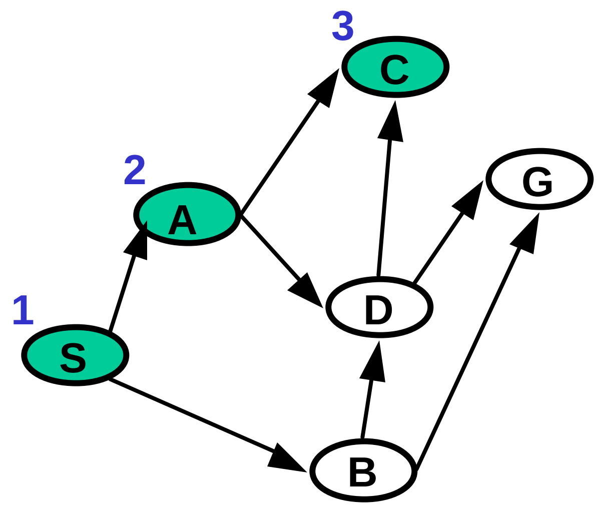

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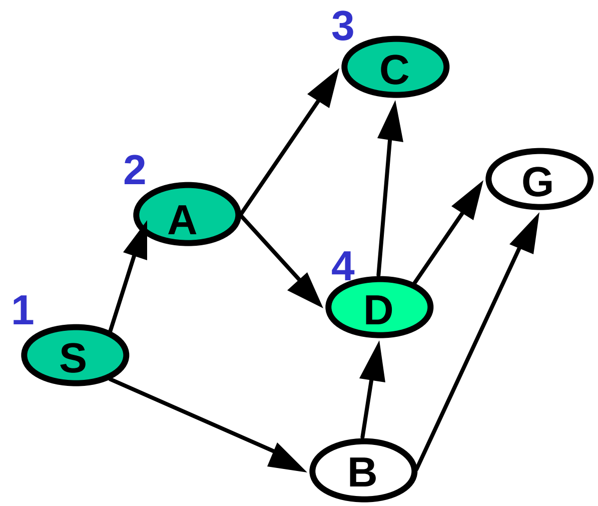
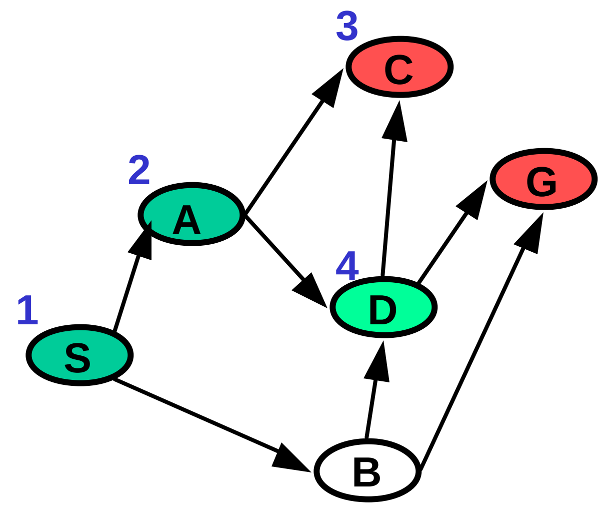


Added paths in blue

Depth-First

Pick first element of Q; Add path extensions to front of Q

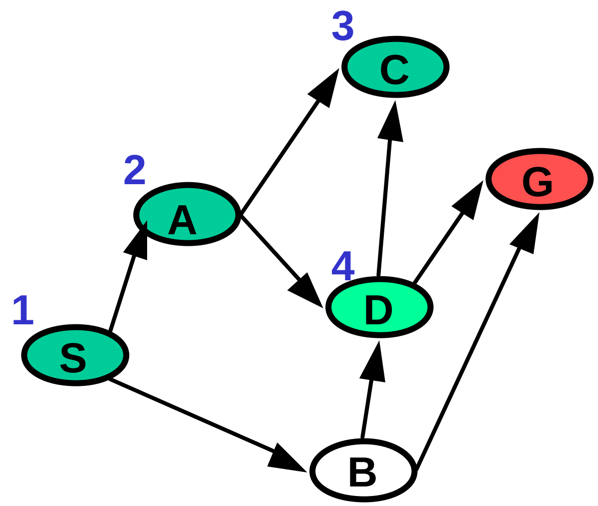
	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	(C D A S)(G D A S) (B S)



Depth-First

Pick first element of Q; Add path extensions to front of Q

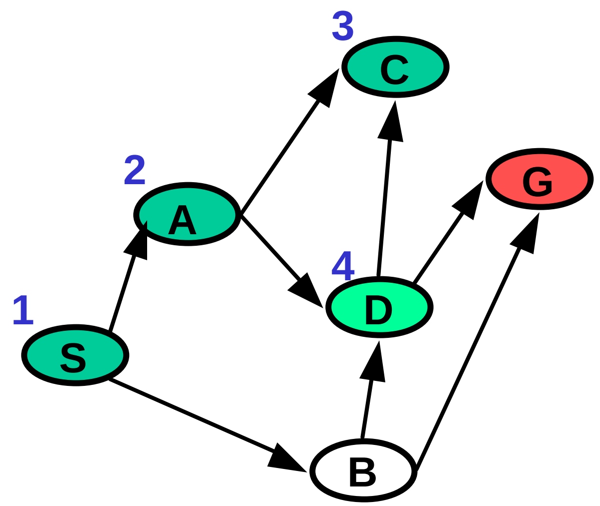
	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	(C D A S)(G D A S) (B S)



Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	(C D A S)(G D A S) (B S)
6	(G D A S)(B S)



Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
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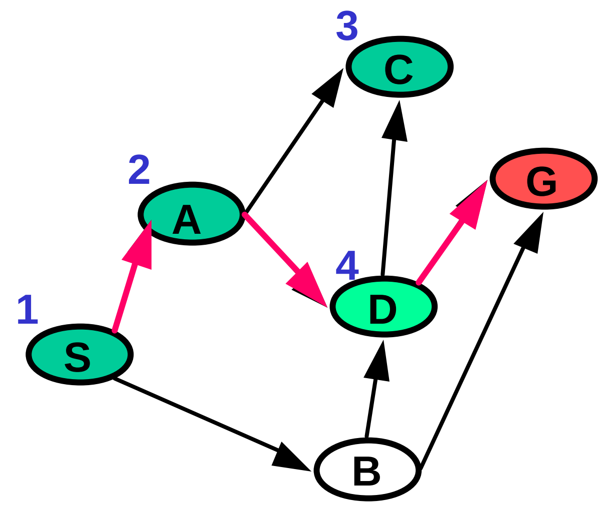

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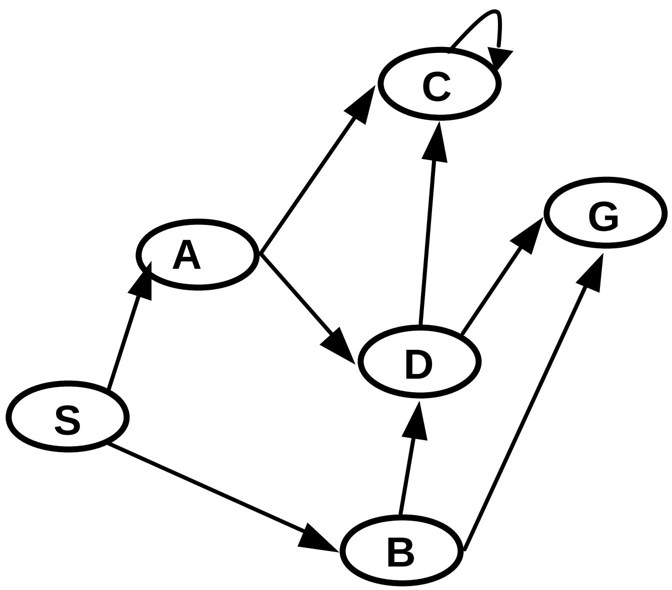
  - A generic search algorithm
  - Depth-first search example
  - Handling cycles
  - Breadth-first search example (do at home)

Issue: Starting at S and moving top to bottom, will depth-first search ever reach G?

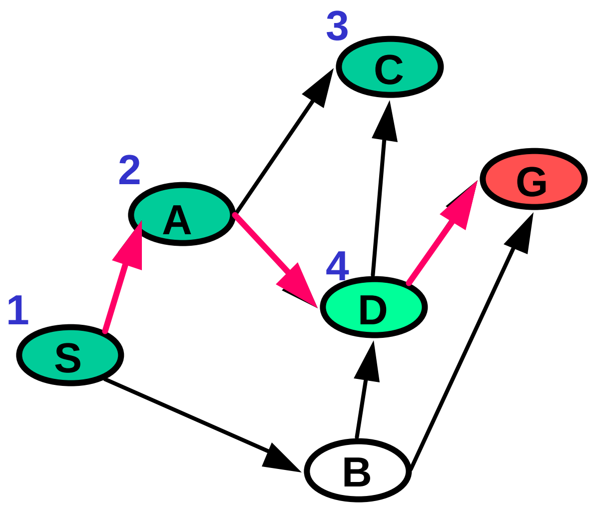


Can effort be wasted in more mild cases?

Depth-First

Pick first element of Q; Add path extensions to front of Q

	Q
1	(S)
2	(A S) (B S)
3	(C A S) (D A S) (B S)
4	(D A S) (B S)
5	(C D A S)(G D A S) (B S)
6	(G D A S)(B S)



- C visited multiple times
- Multiple paths to C, D & G

How much wasted effort can be incurred in the worst case?

How Do We Avoid Repeat Visits?

Idea:

- **Keep track of nodes already visited.**
- **Do not place visited nodes on Q.**

Does it maintain correctness?

- **Any goal reachable from a node that was visited a second time would be reachable from that node the first time.**

Does it always improve efficiency?

- **Guarantees each node appears at most once at the head of a path in Q.**

Simple Search Algorithm

Let Q be a list of partial paths,
Let S be the start node and
Let G be the Goal node.

1. **Initialize Q with partial path (S) as only entry; set Visited = ( )**
2. **If Q is empty, fail. Else, pick some partial path N from Q**
3. **If head(N) = G, return N (goal reached!)**
4. **Else**
   a) **Remove N from Q**
   b) **Find all children of head(N) not in Visited and create all the one-step extensions of N to each child.**
   c) **Add to Q all the extended paths;**
   d) **Add children of head(N) to Visited**
   e) **Go to step 2.**

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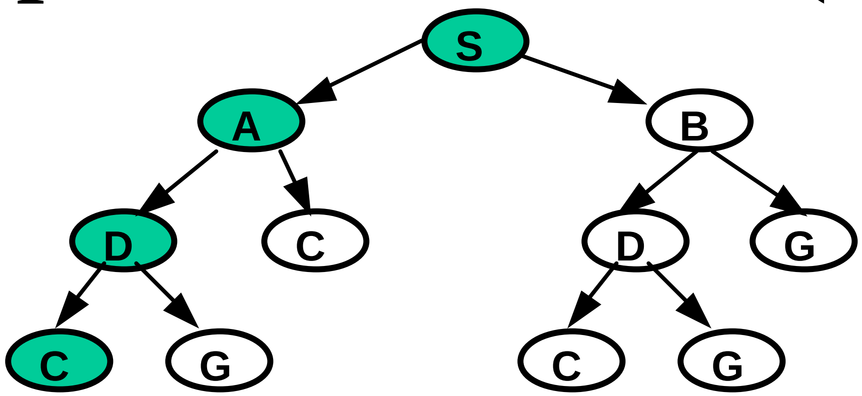
Depth First Search (DFS)

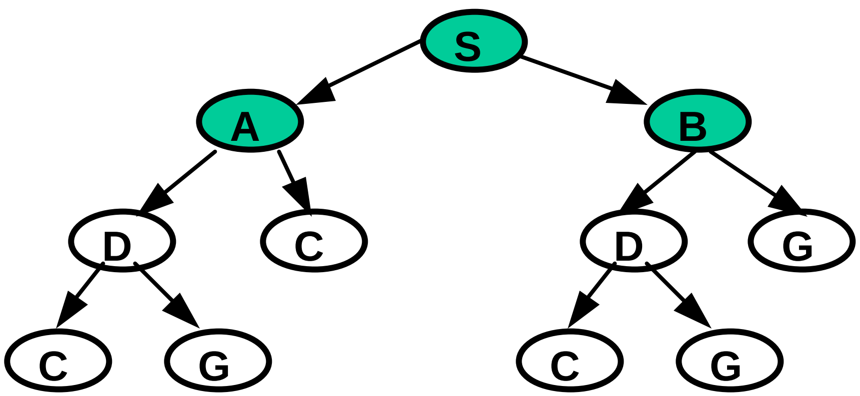


Depth-first:

Add path extensions to **front** of Q

Pick first element of Q

Breadth First Search (BFS)



Breadth-first:

Add path extensions to **back** of Q

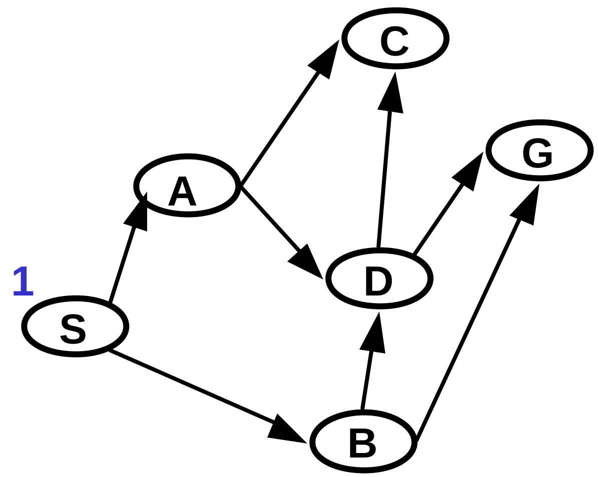
Pick first element of Q

For each search type, where do we place the children on the queue?

Breadth-First

Pick first element of Q; Add path extensions to end of Q

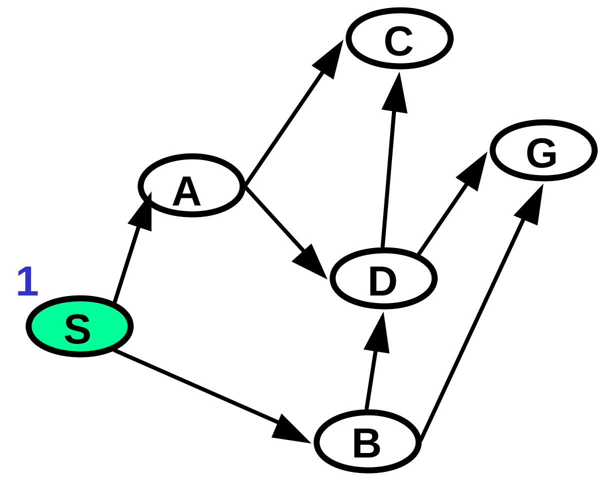
	Q	Visited
1	(S)	S
2		
3		
4		
5		
6		



Breadth-First

Pick first element of Q; Add path extensions to end of Q

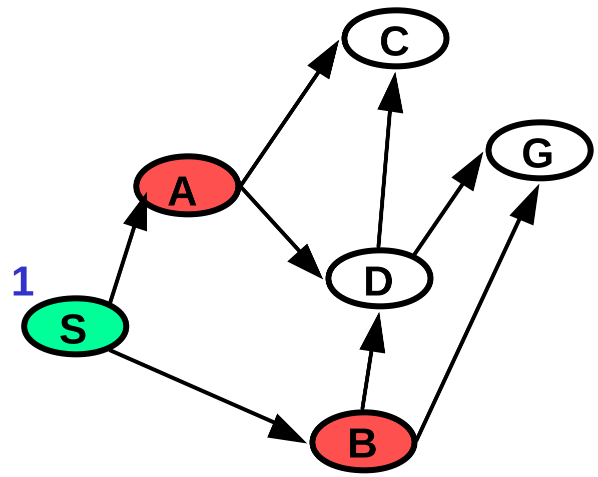
	Q	Visited
1	~~(S)~~	S
2		
3		
4		
5		
6		



Breadth-First

Pick first element of Q; Add path extensions to end of Q

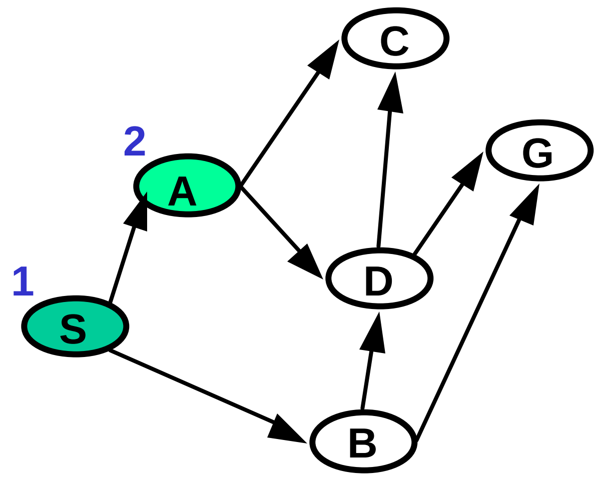
	Q	Visited
1	~~(S)~~	S
2	(A S) (B S)	A,B,S
3		
4		
5		
6		



Breadth-First

Pick first element of Q; Add path extensions to end of Q

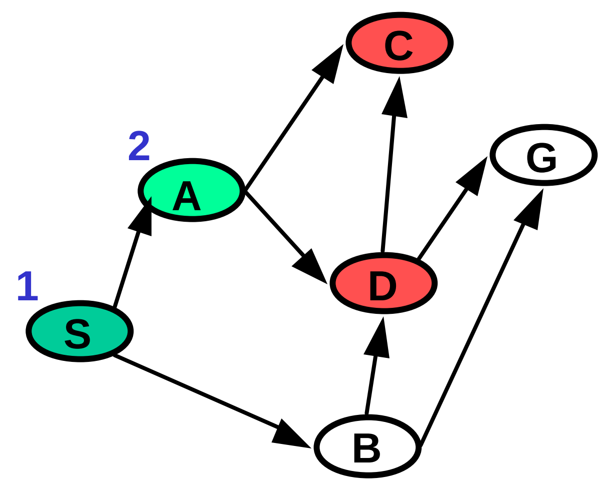
	Q	Visited
1	(S)	S
2	~~(A S)~~ (B S)	A,B,S
3		
4		
5		
6		



Breadth-First

Pick first element of Q; Add path extensions to end of Q

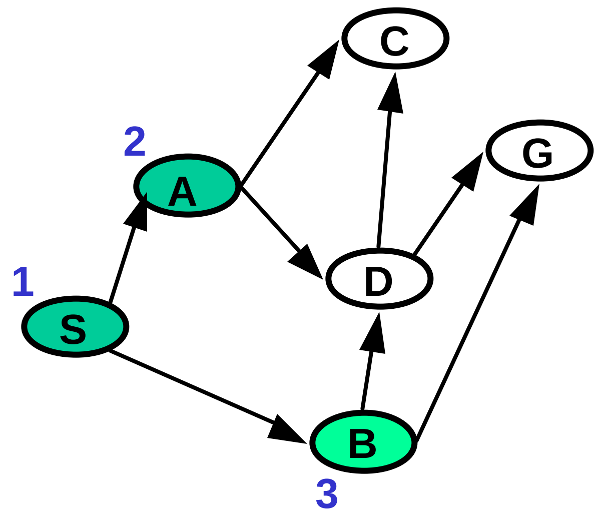
	Q	Visited
1	(S)	S
2	~~(A S)~~ (B S)	A,B,S
3	(B S) (C A S) (D A S)	C,D,B,A,S
4		
5		
6		



Breadth-First

Pick first element of Q; Add path extensions to end of Q

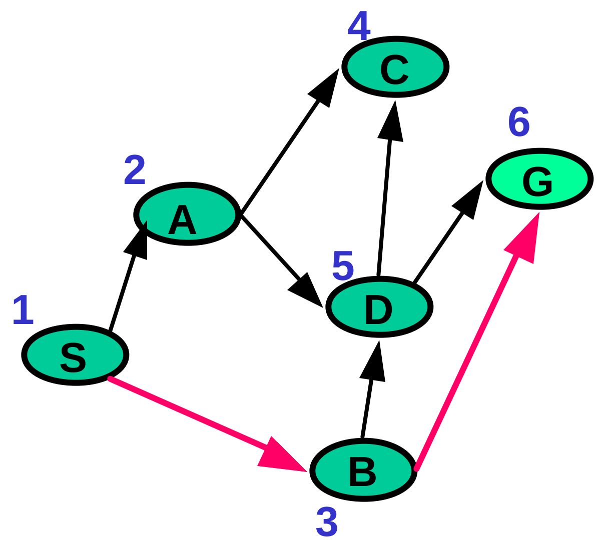
	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3	~~(B S)~~ (C A S) (D A S)	C,D,B,A,S
4		
5		
6		



Breadth-First

Pick first element of Q; Add path extensions to end of Q

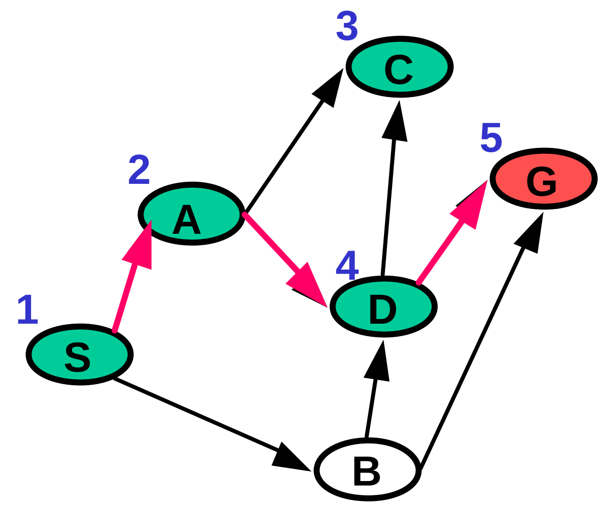
	Q	Visited
1	(S)	S
2	(A S) (B S)	A,B,S
3	(B S) (C A S) (D A S)	C,D,B,A,S
4	(C A S) (D A S) (G B S)*	G,C,D,B,A,S
5	(D A S) (G B S)	G,C,D,B,A,S
6	(G B S)	G,C,D,B,A,S



Depth-first with visited list

Pick first element of Q; Add path extensions to front of Q

	Q	Visited
1	(S)	S
2	(A S) (B S)	A, B, S
3	(C A S) (D A S) (B S)	C,D,B,A,S
4	(D A S) (B S)	C,D,B,A,S
5	(G D A S) (B S)	G,C,D,B,A,S



Summary

- Most problem solving tasks may be encoded as state space search.
- Basic data structures for search are graphs and search trees.
- Depth-first and breadth-first search may be framed,
  among others, as instances of a generic search strategy.
- Cycle detection is required to achieve efficiency and completeness.

Appendix

Breadth-First (without Visited list)

Pick first element of Q; Add path extensions to end of Q

	Q
1	(S)
2	(A S) (B S)
3	(B S) (C A S) (D A S)
4	(C A S) (D A S) (D B S) (G B S)*
5	(D A S) (D B S) (G B S)
6	(D B S) (G B S) (C D A S) (G D A S)
7	(G B S) (C D A S) (G D A S)