16.070

Introduction to Computers & Programming

Data structures: Stack ADT, Queue ADT, (Access types), Linked lists

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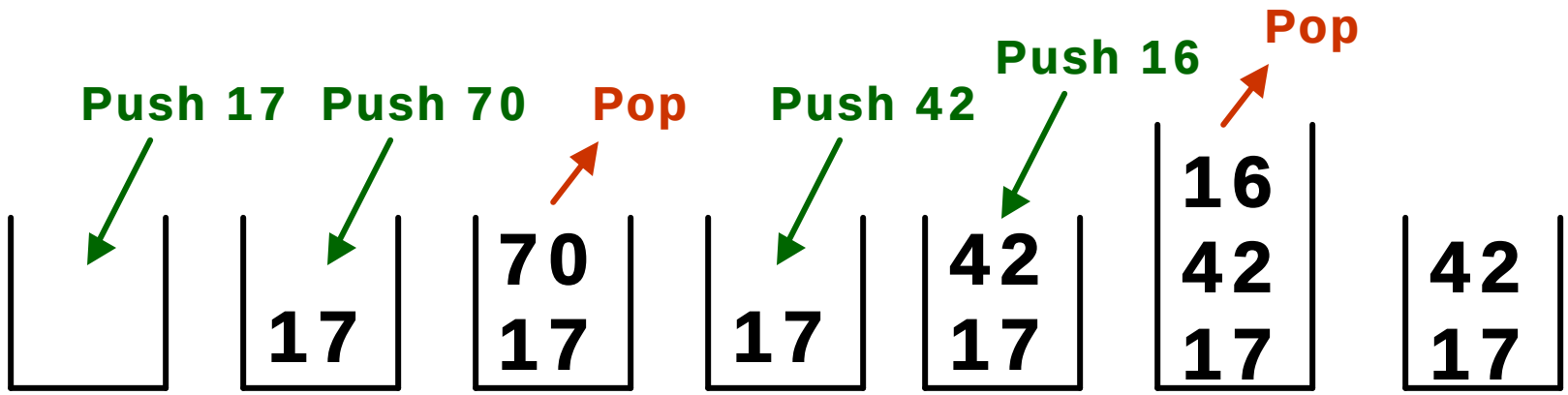
Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction of a data structure
- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations

The Stack ADT

- A classic data structure giving Last-In, First-Out (**LIFO**) access to elements of some type.

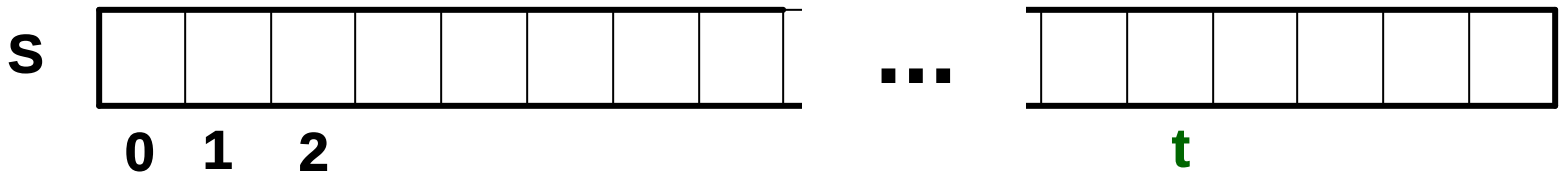
- Main stack operations:
  - **push**(object): inserts an element
  - Object **pop**(): removes and returns the last inserted element
- Auxiliary stack operations:
  - Object **top**(): returns the last inserted element without removing it
  - Integer **size**(): returns the number of elements stored
  - Boolean **isEmpty**(): indicates whether no elements are stored



Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

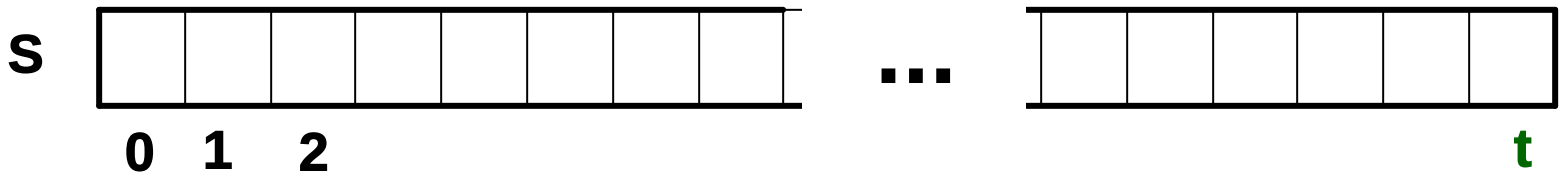
```
Algorithm size()
    return t+1
Algorithm pop()
    if isEmpty() then
        throw EmptyStackExcptn
    else
        t := t-1
        return S[t+1]
```



Array-based Stack

- The array storing the stack elements may become full
- A push operation will then throw an exception

```
Algorithm push(o)
  if t=S'length-1 then
    throw FullStackException
  else
    t := t + 1
    S[t] := o
```



Performance and Limitations

- Performance
  - Let N be the number of elements in the stack
  - The space used is O(N)
  - Each operation used is O(1)
- Limitations
  - The maximum size of the stack must be defined a priori and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception

Stack Implementations

- Array-based
  - Items “pushed” onto stack are added at next available array location. Requires **IsFull** operation.
  - Top of stack is referenced by an integer index into the array, which contains the index of the most recently pushed item.
- Linked-List
  - Items “pushed” onto stack stored in dynamically-allocated nodes added to the front (head) of the list.
  - Top of stack is referenced by the head pointer.
- Inheritance
  - Stack class is derived from a List class
  - Internal representation depends on the internals of the List
  - For linked-lists, List must provide equivalent of **InsertFront**

Some Stack Applications

- Direct applications
  - Postfix Expression Evaluation in RPN calculators
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

Infix vs. Postfix

Infix Expressions:	Corresponding Postfix:
`5 + 3 + 4 + 1`	`5 3 + 4 + 1 +`
`(5 + 3) * 10`	`5 3 + 10 *`
`(5 + 3) * (10 - 4)`	`5 3 + 10 4 - *`
`5 * 3 / (7 - 8)`	`5 3 * 7 8 - /`
`(b*b - 4*a*c) / (2*a)`	`b b * 4 a * c * - 2 a * /`

How to Evaluate Postfix

A program can evaluate postfix expressions by reading the expression from left to right and following these simple rules:

- if a number is read, push it on the stack
- if an operator is read, pop two numbers off the stack (the first number popped is the *second* binary operand)
- apply the operation to the numbers, and push the result back onto the stack
- when the expression is complete, the number on top of stack is the answer

How is a bad postfix expression indicated?

Evaluating infix expressions

- Need 2 stacks. 1 numbers, 1 operators

```
while tokens available
      if (number) push on number_stack
      if (operator) push on operator_stack
      if (‘(‘) do nothing

      else                  --must be ‘)’
            pop 2 numbers and 1 operator
            calculate
            push result on number_stack
end
```

The Queue ADT

- The Queue ADT stores arbitrary objects
- Insertions and deletions follow the first-in first-out scheme (FIFO)
- Insertions are at the rear of the queue and removals are at the front of the queue
- Main queue operations:
  - enqueue(object): inserts an element at the end of the queue
  - object dequeue(): removes and returns the element at the front of the queue

The Queue ADT

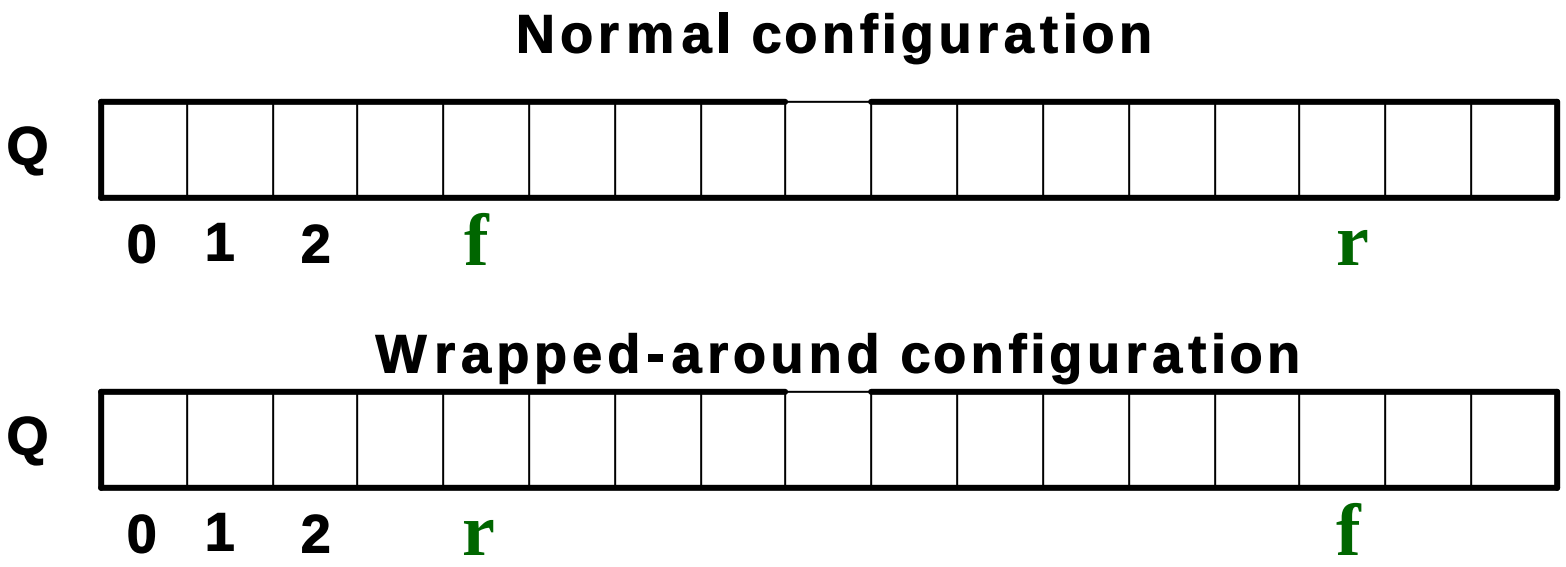
- Auxiliary queue operations:
  - object front(): returns the element at the front without removing it
  - integer size(): returns the number of elements stored
  - boolean isEmpty(): indicates whether no elements are stored
- Exceptions
  - Attempting the execution of dequeue or front on an empty queue throws an EmptyQueueException

Applications of Queues

- Direct applications
  - Waiting lists (bureaucracy)
  - Access to shared resources (e.g., printer)
- Indirect applications
  - Auxiliary data structures for algorithms
  - Component of other data structures

Array-based Queue

- Use an array of size **N** in a circular fashion
- Two variables keep track of the front and rear
  - **f** index of the front element
  - **r** index immediately past the rear element
- Array location **r** is kept empty

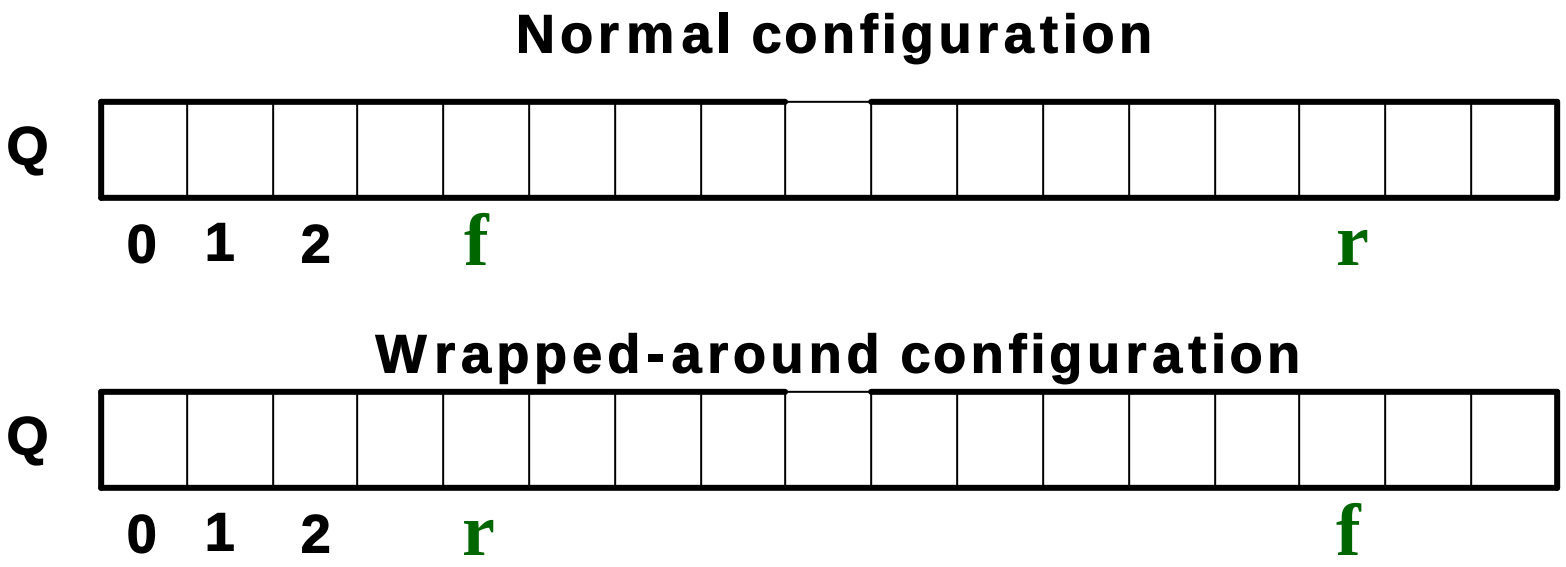


Queue Operations

- We use the modulo operator

```
Algorithm size()
    return (N-f+r) mod N

Algorithm isEmpty()
    return (f=r)
```

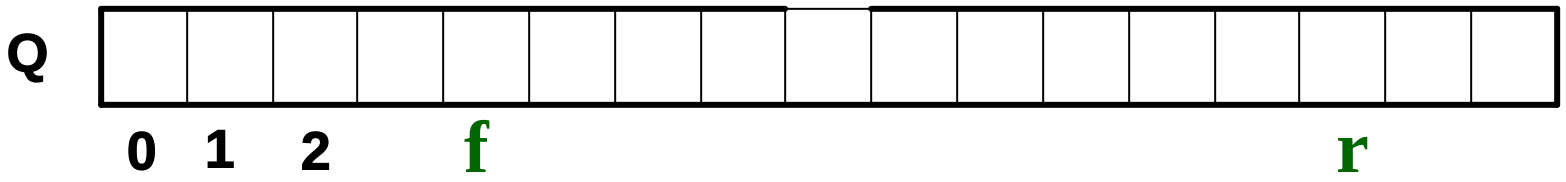


Queue Operations

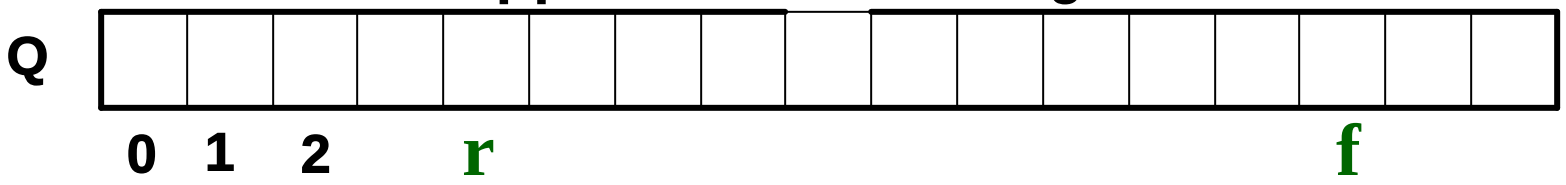
- Operation enqueue throws an exception if the array is full
- This exception is implementation dependent

```
Algorithm enqueue(o)
  if size() = N-1 then
    throw FullQException
  else
    Q[r] := o
    r := (r+1) mod N
```

Normal configuration



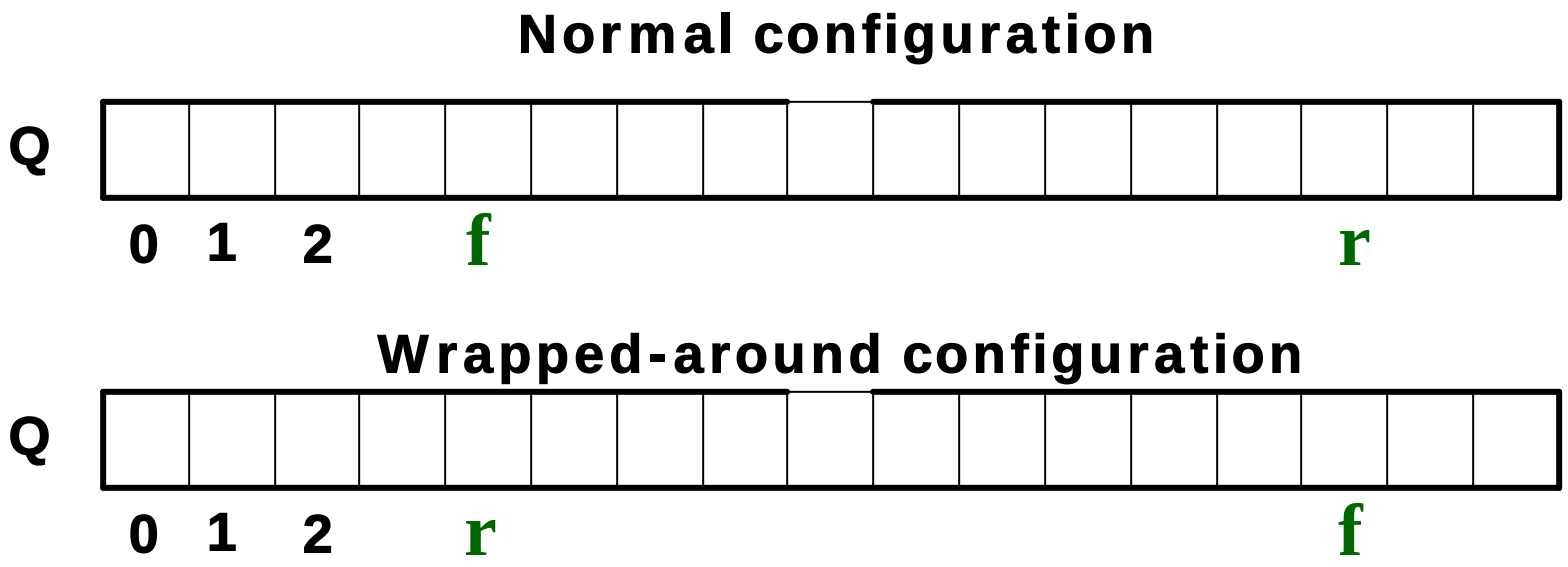
Wrapped-around configuration



Queue Operations

- Operation dequeue throws an exception if the queue is empty
- This exception is specified in the queue ADT

```
Algorithm dequeue()
    if isEmpty() then
        throw EmptyQException
    else
        o := Q[f]
        f := (f+1) mod N
    return o
```

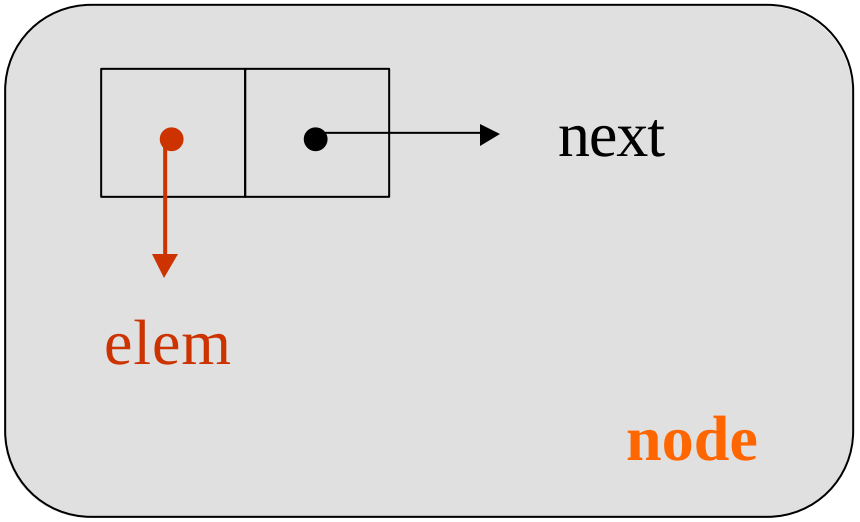


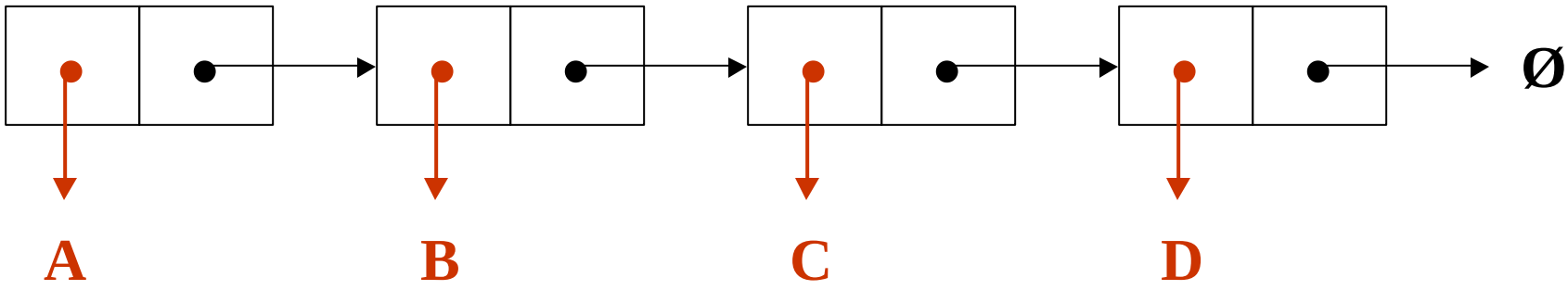
Growable Array-based Queue

- In an enqueue operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one

Singly Linked List

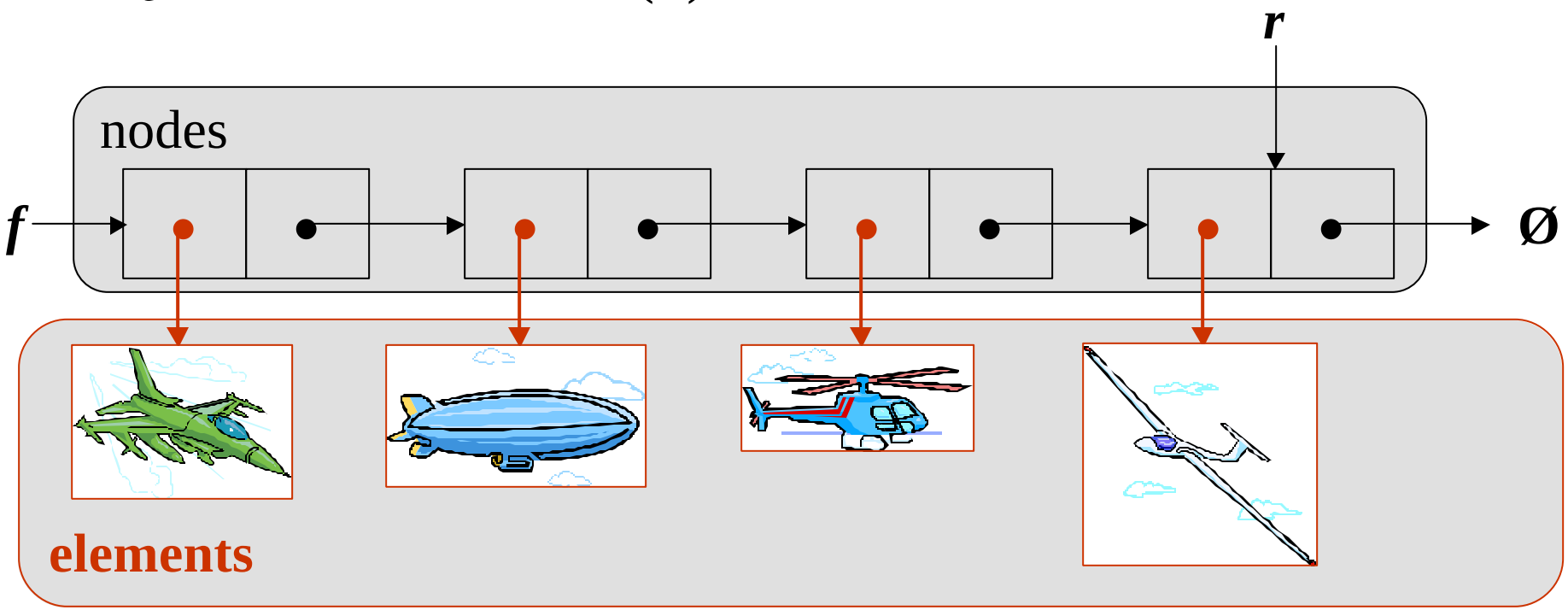
- A singly linked list is a concrete data structure consisting of a sequence of nodes
- Each node stores
  - Element
  - Link to the next node





Queue with a Singly Linked List

- We can implement a queue with a singly linked list
  - The front element is stored at the first node
  - The rear element is stored at the last node
- The space used is O(n) and each operation of the Queue ADT takes O(1) time

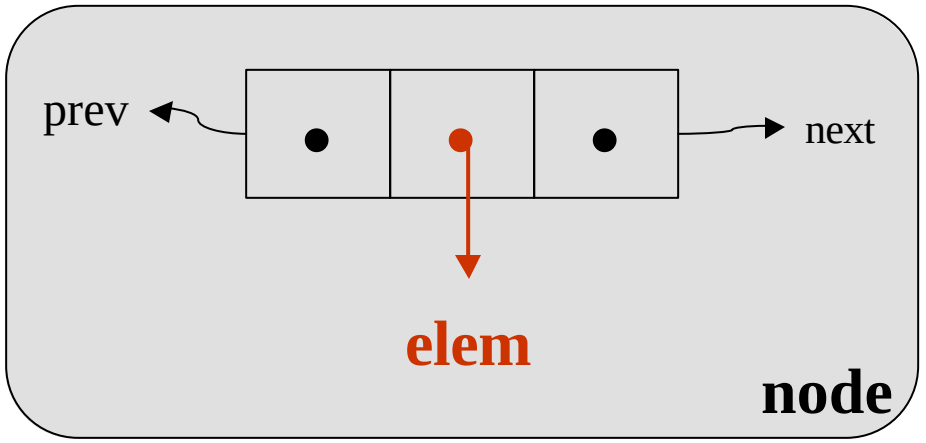


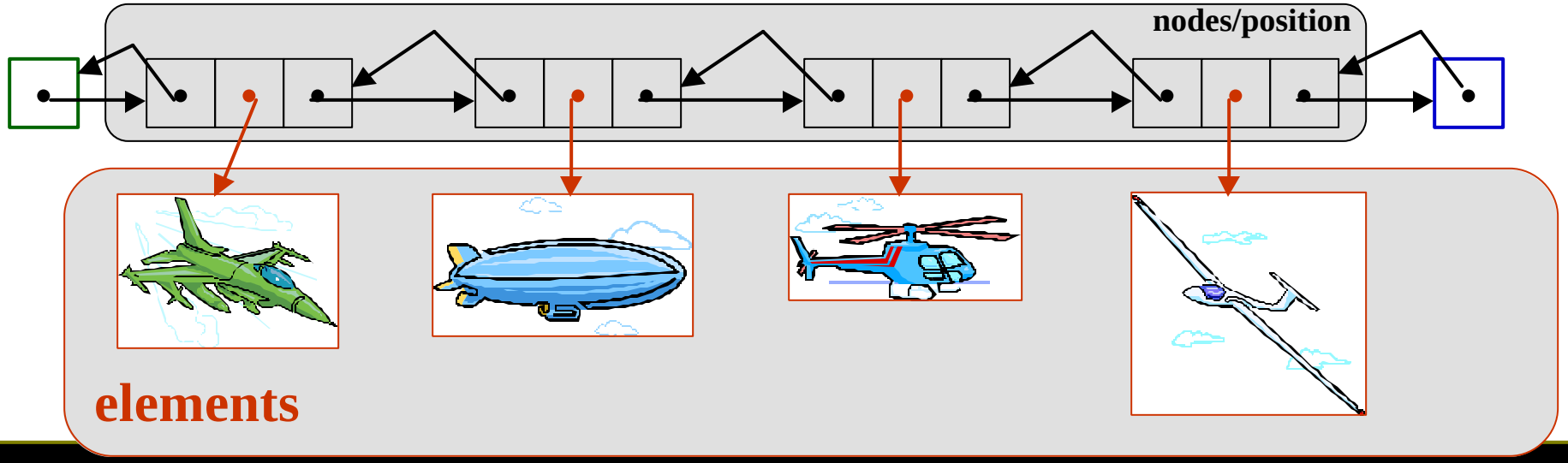
List ADT

- The list ADT models a sequence of positions storing arbitrary objects
- It allows for insertion and removal in the “middle”
- Query methods:
  - isFirst(p), isLast(p)
- Accessor methods:
  - first(), last()
  - before(p), after(p)
- Update methods:
  - replaceElement(p, o), swapElements(p, q)
  - insertBefore(p, o), insertAfter(p, o)
  - insertFirst(o), insertLast(o)
  - remove(p)

Doubly Linked List

- A doubly linked list provides a natural implementation of the List ADT
- Nodes implement Position and store:
  - element
  - link to the previous node
  - link to the next node
- Special trailer and header node





Analysis of Merge Sort

Statement	Effort
`MergeSort(A, lower_bound, upper_bound)`	
`begin`	`T(n)`
`  if (lower_bound < upper_bound)`	Q`(1)`
`  mid = (lower_bound + upper_bound)/ 2`	Q`(1)`
`  MergeSort(A, lower_bound, mid)`	`T(n/2)`
`  MergeSort(A, mid+1, upper_bound)`	`T(n/2)`
`  Merge(A, lower_bound, mid,upper_bound)`	Q`(n)`
`end`	

- So T(n) = $\Theta(1)$ when n = 1, and

  $2T(n/2) + \Theta(n)$ when n > 1

- So what (more succinctly) is T(n)?

Recurrences

- The expression:

$$T(n) = \begin{cases} c & n = 1 \\ 2T\left(\frac{n}{2}\right) + c & n > 1 \end{cases}$$

  is a *recurrence*.

  - Recurrence: an equation that describes a function in terms of its value on smaller functions