Module 8

General Beam Theory

Learning Objectives

- Generalize simple beam theory to three dimensions and general cross sections
- Consider combined effects of bending, shear and torsion
- Study the case of shell beams

8.1 Beams loaded by transverse loads in general directions

Readings: BC 6

So far we have considered beams of fairly simple cross sections (e.g. having symmetry planes which are orthogonal) and transverse loads acting on the planes of symmetry. Figure 8.1 shows examples of beams loaded on a plane which does not coincide with a plane of symmetry of its cross section.

In this section, we will consider beams with cross section of arbitrary shape which are loaded on planes that do not in general coincide with symmetry planes (or as we will see later more precisely, with principal directions of inertia of the cross section).

We will still adopt Euler-Bernoulli hypothesis, which implies that the kinematic assumptions about the allowed deformation modes of the beam remain the same, see Section 7.1.1.

The displacement field is still given by equations (**??**), whereas the strain field is given by equations (**??**). It should be noted that the origin of coordinates in the cross section is still unspecified.

8.1.1 Constitutive law for the cross section

We will assume that the beam is made of linear elastic isotropic materials and use Hooke's law. Since the strain distribution is still bound by the sames constraints, the stress distribution will be as before:

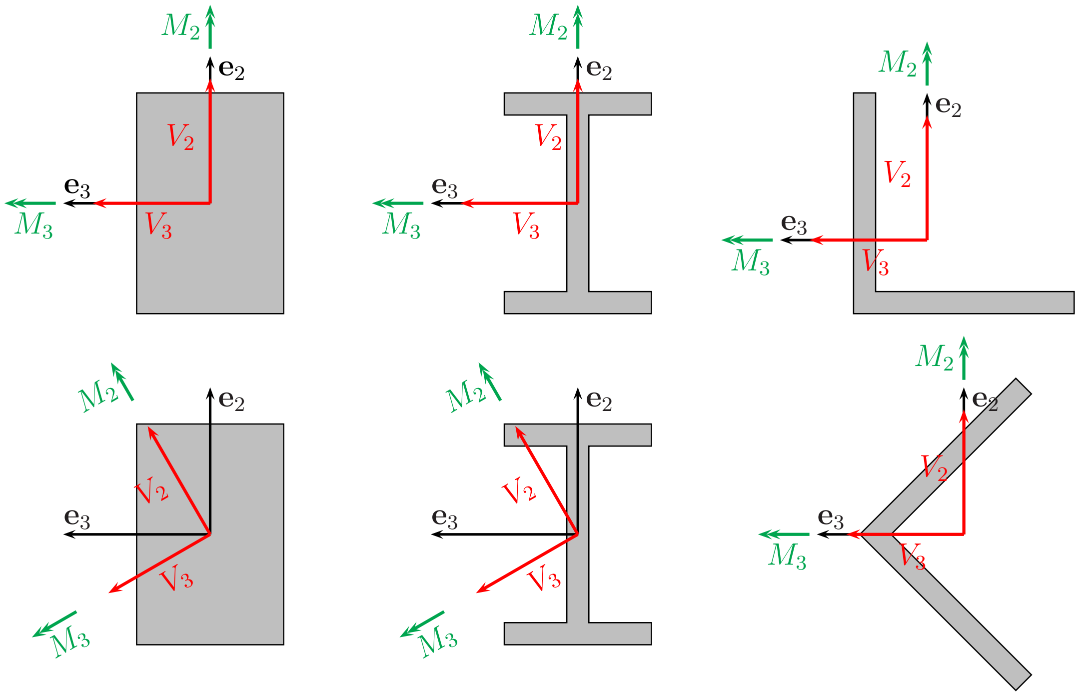

Figure 8.1: Loading of beams in general planes and somewhat general cross sections

$$\sigma_{11}(x_1, x_2, x_3) = E\epsilon_{11}(x_1, x_2, x_3) = E[\bar{u}_1'(x_1) - x_2\bar{u}_2''(x_1) - x_3\bar{u}_3''(x_1)] \tag{8.1}$$

Following with the by now usual plan to build a structural theory, we proceed to compute the resultants:

Axial force N_1

$$N_1(x_1) = \int_A \sigma_{11}(x_1, x_2, x_3)dA$$
$$= \underbrace{\left[\int_A E dA\right]}_{S} \bar{u}_1'(x_1) - \underbrace{\left[\int_A E x_2 dA\right]}_{S_2} \bar{u}_2''(x_1) - \underbrace{\left[\int_A E x_3 dA\right]}_{S_3} \bar{u}_3''(x_1)$$
$$\boxed{N_1(x_1) = S\bar{u}_1'(x_1) - S_2\bar{u}_2''(x_1) - S_3\bar{u}_3''(x_1)} \tag{8.2}$$

where S is the modulus-weighted area or axial stiffness, S_2, S_3 are respectively the modulus-weighted first moments of area of the cross section with respect to the $\mathbf{e}_3$ and $\mathbf{e}_2$ axes.

Bending moments $M_2(x_1), M_3(x_1)$

$$M_2(x_1) = \int_A \sigma_{11} x_3 dA =$$

$$= \underbrace{\left[\int_A E x_3 dA\right]}_{S_3} \bar{u}_1'(x_1) - \underbrace{\left[\int_A E x_2 x_3 dA\right]}_{H_{23}} \bar{u}_2''(x_1) - \underbrace{\left[\int_A E x_3^2 dA\right]}_{H_{22}} \bar{u}_3''(x_1)$$

$$\boxed{M_2(x_1) = S_3 \bar{u}_1'(x_1) - H_{23} \bar{u}_2''(x_1) - H_{22} \bar{u}_3''(x_1)} \quad (8.3)$$

$$M_3(x_1) = -\int_A \sigma_{11} x_2 dA =$$

$$= -\underbrace{\left[\int_A E x_2 dA\right]}_{S_2} \bar{u}_1'(x_1) + \underbrace{\left[\int_A E x_2^2 dA\right]}_{H_{33}} \bar{u}_2''(x_1) + \underbrace{\left[\int_A E x_3 x_2 dA\right]}_{H_{23}} \bar{u}_3''(x_1)$$

$$\boxed{M_3(x_1) = -S_2 \bar{u}_1'(x_1) + H_{33} \bar{u}_2''(x_1) + H_{23} \bar{u}_3''(x_1)} \quad (8.4)$$

We note that we have used some of the previously defined section stiffness coefficients S, H_{33}, but we have also introduced some new ones. Summarizing all:

Area:	$S = \int_A E dA$
First moment of area wrt $\mathbf{e}_3$	$S_2 = \int_A E x_2 dA$
First moment of area wrt $\mathbf{e}_2$	$S_3 = \int_A E x_3 dA$
Second moment of area wrt $\mathbf{e}_3$	$H_{22} = \int_A E x_3^2 dA$
Second moment of area wrt $\mathbf{e}_2$	$H_{33} = \int_A E x_2^2 dA$
Second cross moment of area wrt $\mathbf{e}_2, \mathbf{e}_3$	$H_{23} = \int_A E x_2 x_3 dA$

Table 8.1: Modulus-weighted cross section stiffness coefficients

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Concept Question 8.1.1. Give an interpretation to the various cross section stiffness coefficients by observing the "strains" and resultant forces they relate

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The main conclusion from this general beam theory is that there is a coupling among all stress resultants and all "strain measures". Specifically, this means that a curvature in one plane can cause not only a bending moment in the respective plane but also a moment in the plane orthogonal to it as well as an axial force. Also, that the axial strain $\bar{u}_1'$ can cause moments in both orthogonal planes.

A first simplification of these expressions is obtained if we first find the *modulus-weighted centroid* of the cross section x_2^c, x_3^c and then refer all our quantities with respect to that point (i.e. place the origin of our axes from where we measure x_2, x_3 at that point). In that case, as we saw before:

$$x_2^c = \frac{\overbrace{\int_A E x_2 dA}^{S_2}}{\underbrace{\int_A E dA}_{S}} = 0, x_3^c = \frac{\overbrace{\int_A E x_3 dA}^{S_3}}{\underbrace{\int_A E dA}_{S}} = 0, \tag{8.5}$$

and the coupling between axial and flexural quantities disappears, i.e. the sectional constitutive equations become:

$$\boxed{N_1(x_1) = S\bar{u}_1'(x_1)} \tag{8.6}$$

$$\boxed{M_2(x_1) = -H_{23}^c \bar{u}_2''(x_1) - H_{22}^c \bar{u}_3''(x_1)} \tag{8.7}$$

$$\boxed{M_3(x_1) = +H_{33}^c \bar{u}_2''(x_1) + H_{23}^c \bar{u}_3''(x_1)} \tag{8.8}$$

Note that we have also added the superscript $()^c$ to the stiffness coefficients to make it clear that now these quantities need to be evaluated using as the origin the modulus weighted centroid.

In many cases we know the moments and axial force and we are interested in finding the internal stresses and beam deflections. This requires inverting the above relations:

$$\bar{u}_1'(x_1) = \frac{1}{S} N_1(x_1) \tag{8.9}$$

$$\bar{u}_2''(x_1) = \frac{H_{23}^c}{\Delta_H} M_2(x_1) + \frac{H_{22}}{\Delta_H} M_3(x_1) \tag{8.10}$$

$$\bar{u}_3''(x_1) = -\frac{H_{33}^c}{\Delta_H} M_2(x_1) - \frac{H_{23}}{\Delta_H} M_3(x_1) \tag{8.11}$$

With $\Delta_H = H_{22}^c H_{33}^c - H_{23}^c H_{23}^c$.

The stresses can then be written as:

$$\sigma_{11} = E\left[\frac{N_1}{S} + x_3 \frac{H_{33}^c M_2 + H_{23}^c M_3}{\Delta_H} - x_2 \frac{H_{23}^c M_2 + H_{22}^c M_3}{\Delta_H}\right] \tag{8.12}$$

which can be rearranged in a more useful form as:

$$\sigma_{11} = E\left[\frac{N_1}{S} - \frac{x_2 H_{23}^c - x_3 H_{33}^c}{\Delta_H} M_2 - \frac{x_2 H_{22}^c - x_3 H_{23}^c}{\Delta_H} M_3\right] \tag{8.13}$$

8.1.2 Equilibrium equations

The equilibrium equations for the general beam theory we are developing will be derived with the same considerations as we did in Section 7.3.2 with two modifications: 1) addition of equilibrium of moments in the $\mathbf{e}_2$ direction, 2) contribution of the axial force. Figures 8.2(a) and 8.2(b) show a free-body diagram of a beam slice subjected to both axial and transverse loads in two orthogonal but otherwise arbitrary directions (i.e, the loading direction does not necessarily match the principal axis of the cross section of the beam). The internal and external loads are shown in preparation for enforcing equilibrium.

From figure 8.2(a) we obtain the following relations for the axial N_1 and shear V_2 forces, and the bending moment M_3 in the $(\mathbf{e}_1,\mathbf{e}_2)$ plane:

$$\begin{cases} \dfrac{dN_1}{dx_1} = -p_1(x_1) \\ \\ \dfrac{dV_2}{dx_1} = -p_2(x_1) \\ \\ \dfrac{dM_3}{dx_1} + V_2 = x_{2a}p_1(x_1) \end{cases} \tag{8.14}$$

From figure 8.2(b) we obtain the following relations for the axial N_1 and shear V_3 forces, and the bending moment M_2 in the $(\mathbf{e}_1,\mathbf{e}_3)$ plane:

$$\begin{cases} \dfrac{dN_1}{dx_1} = -p_1(x_1) \\ \\ \dfrac{dV_3}{dx_1} = -p_3(x_1) \\ \\ \dfrac{dM_2}{dx_1} - V_3 = -x_{3a}p_1(x_1) \end{cases} \tag{8.15}$$

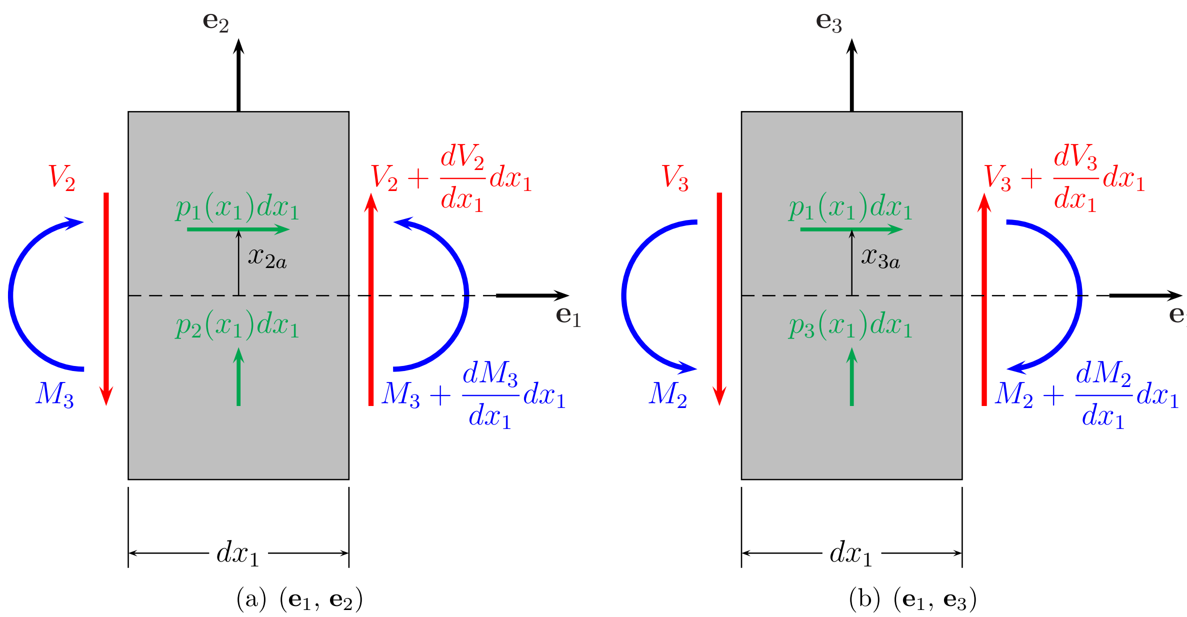


Figure 8.2: Equilibrium in both, $(\mathbf{e}_1,\mathbf{e}_2)$ and $(\mathbf{e}_1,\mathbf{e}_3)$ planes of a beam slice subjected to axial and transverse loads in general directions.

These equations can be combined by differentiating the moment equations and replacing the shear force equations in them:

$$\begin{aligned}
\frac{d^2 M_3}{dx_1^2} &= \frac{d}{dx_1}\left(-V_2 + x_{2a} p_1(x_1)\right) \\
&= -\frac{dV_2}{dx_1} + \frac{d}{dx_1}\left(x_{2a} p_1(x_1)\right) \\
&= p_2(x_1) + \frac{d}{dx_1}\left(x_{2a} p_1(x_1)\right)
\end{aligned}$$

$$\begin{aligned}
\frac{d^2 M_2}{dx_1^2} &= \frac{d}{dx_1}\left(V_3 - x_{3a} p_1(x_1)\right) \\
&= \frac{dV_3}{dx_1} - \frac{d}{dx_1}\left(x_{3a} p_1(x_1)\right) \\
&= -p_3(x_1) - \frac{d}{dx_1}\left(x_{3a} p_1(x_1)\right)
\end{aligned}$$

To summarize, the two equilibrium equations are:

$$\boxed{\begin{aligned}
\frac{d^2 M_2}{dx_1^2} &= -p_3(x_1) - \frac{d}{dx_1}\left(x_{3a} p_1(x_1)\right) \\
\frac{d^2 M_3}{dx_1^2} &= p_2(x_1) + \frac{d}{dx_1}\left(x_{2a} p_1(x_1)\right)
\end{aligned}} \tag{8.16}$$

The main peculiarity in these equations is the appearance of the terms involving the axial distributed force p_1 multiplied by the operative moment arm. This is a direct result of the fact that we cannot assume *a priori* that this force will be applied at the modulus-weighted centroid and may, thus, produce a contribution to the bending moment.

8.1.3 Governing equations

Replacing the sectional constitutive laws from Section 8.1.1 into the equations from the previous section, we obtain the governing equations:

$$\begin{cases}
(S\overline{u}_1')' = -p_1 \\
\\
(H_{33}^c \overline{u}_2'' + H_{23}^c \overline{u}_3'')'' = p_2 + (x_{2a} p_1)' \\
(H_{23}^c \overline{u}_2'' + H_{22}^c \overline{u}_3'')'' = p_3 + (x_{3a} p_1)'
\end{cases} \tag{8.17}$$

Concept Question 8.1.2. Observe the governing equations and try to answer the following questions:

1. What is the main difficulty in solving these equations compared to simple beam theory?

2. Can you think of any situations in which the solution of the fourth order coupled system of ODEs can be avoided?

Boundary conditions When the system has to be solved, appropriate boundary conditions must be provided. Depending on the type of idealization of the physical system, type of support and loading, we can have a combination of imposed displacements, constrained rotations, forces or moments, i.e.

$$\overline{u}_1 = \overline{u}_2 = \overline{u}_3 = 0 \qquad \text{and} \qquad \overline{u}_2' = \overline{u}_3' = 0 \tag{8.18}$$

$$\begin{cases} N_1 = P_1 \\ V_2 = P_2 \ , \ V_3 = P_3 \\ M_3 = -x_{2a} P_1 \ , \ M_2 = x_{3a} P_1 \end{cases} \tag{8.19}$$

These can be written as a function of derivatives of the beam deflections. $\overline{u}_1$, $\overline{u}_2$ and $\overline{u}_3$:

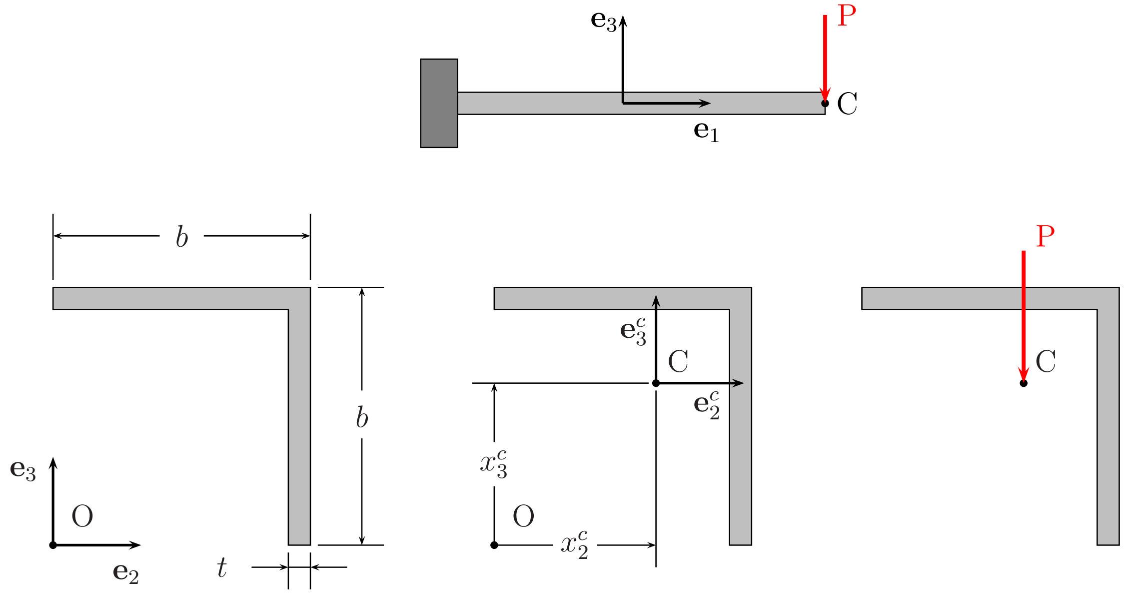


Figure 8.3: Cantilever beam with a L-shaped cross section.

Concept Question 8.1.3. *bending of a beam with a L-shaped cross section.* Let us consider a cantilever beam with a L-shaped cross section as depicted in Figure 8.3. It is assumed that the beam is made of a linear homogeneous material, in this context fully described by its Young's modulus $E = 2 \times 10^{11}$ GPa. The cross section of the beam is 0.1 m wide and high (b); its thickness, t, is equal to 2 mm, and, its length, l, is equal to 2 m. A load P of 200 N is applied at the free-end of the beam, more precisely at C, it modulus-weighted centroid.

1. Compute the coordinates (x_2^c, x_3^c) of the modulus weighted centroid of the section with respect to the origin O.
2. Compute the bending stiffnesses in the coordinate system (x_2^c, x_3^c).
3. Compute the maximum tensile and compressive stresses in the L-shaped cross section.
4. Determine the neutral axis orientation with respect to $\mathbf{e}_2$

8.1.4 Decoupling the problem

In section 8.1.1 we wrote both, the axial force N_1 and the bending moments M_2 and M_3 as a function of the axial and bending sectional stiffnesses $S, S_2, S_3, H_{22}, H_{33}, H_{23}$. These relations were simplified if we referred all our coordinates to the modulus-weighted centroid of the cross section, in which case $S_2 = 0$ and $S_3 = 0$). From the equations of equilibrium obtained in section 8.1.2 we obtain the following matrix system:

$$\left\{ \begin{array}{c} N_1(x_1) \\ M_2(x_1) \\ M_3(x_1) \end{array} \right\} = \left[\begin{array}{ccc} S & 0 & 0 \\ 0 & H_{22}^c & -H_{23}^c \\ 0 & -H_{23}^c & H_{33}^c \end{array} \right] \left\{ \begin{array}{c} \overline{u}_1'(x_1) \\ -\overline{u}_3''(x_1) \\ \overline{u}_2''(x_1) \end{array} \right\} \tag{8.20}$$

Here, we have a partially uncoupled problem. Indeed, the axial force is only related to the first derivative of the displacement along the $\mathbf{e}_1$ direction but the displacement components u_2 and u_3 are coupled because of the presence of the non-zero cross bending stiffness H_{23}. In order to solve the partially uncoupled problem, the main idea is to determine the directions that the axis of the beam should match in order to the problem to be fully uncoupled. In other words, we want the matrix in equation 8.20 to be diagonal, without any coupling term which leads to:

$$H_{23}^c = \int_{A(x_1)} E x_2 x_3 dA = 0 \tag{8.21}$$

which also defines the *principal centroidal axes of bending*. For that purpose, we determine the reference frame (denoted with a $*$ in the following) where the matrix is diagonal, also well-known as principal directions, and define the components of the diagonal matrix which are the principal/eigen values. Of note, the obtained diagonal matrix will satisfy the two equilibrium relations in equation 8.16.

Concept Question 8.1.4. *Decoupled constitutive laws.* Let's consider the associated fully decoupled problem where the matrix in equation 8.20 is diagonal and written as a function of S^*, H_{22}^{c*} and H_{33}^{c*} as follows:

$$\left[\begin{array}{ccc} S^* & 0 & 0 \\ 0 & H_{22}^{c*} & 0 \\ 0 & 0 & H_{33}^{c*} \end{array} \right] \tag{8.22}$$

Write the constitutive laws (expression of axial stress distribution σ_{11}) for this fully decoupled problem as a function of S^*, H_{22}^{c*} and H_{33}^{c*}.

Concept Question 8.1.5. *Decoupled governing equations.* For the same fully decoupled problem as above, write the three governing equations as a function of S^*, H_{22}^{c*} and H_{33}^{c*}.

Concept Question 8.1.6. *Principal centroidal axes of bending.* We consider the fully decoupled problem associated with the diagonal matrix in equation 8.22. Herein, the centroidal axes of bending, also defined as the reference frame, correspond to the principal direction of the diagonal matrix. In this exercise we want to determine both the principal directions and the eigen values S^*, H_{22}^{c*} and H_{33}^{c*}.

Show that $S^* = S$ and that leads to diagonalize a 2×2 matrix you specify.

Using either the general formulae of the diagonalization of a 2×2 matrix or the Mohr's circle relations, to define an expression for both the eigen values and the principal directions.

To summarize, solving a three-dimensional general beam problem consists in decoupling the problem in three separate problems by expressing the compatibility equations, the constitutive laws and the governing equations in the reference frame characterized by the principal centroidal axes of bending. For that purpose, we follow the steps listed below:

- (i) Compute the centroid of the section using the equation 8.5
- (ii) Compute the bending stiffnesses in this axis system using the relations in the table 8.1
- (iii) Compute the orientation of the principal axes of bending using the equation **??**
- (iv) Compute the principal bending stiffnesses using equation **??**

8.2 Bending, shearing and torsion of shell beams

Readings: BC Chapter 8

Aircraft and also some space structures are designed and built using a so-called *semi-monocoque* structural concept. This essentially means that the structure is made of a shell with stiffeners, as shown in Figure 8.4

We will idealize this section by assuming that (see Figure 8.5):

- the flanges and stringers carry only axial stresses σ_{11}
- the skins and webs carry only shear stresses σ_{1s}

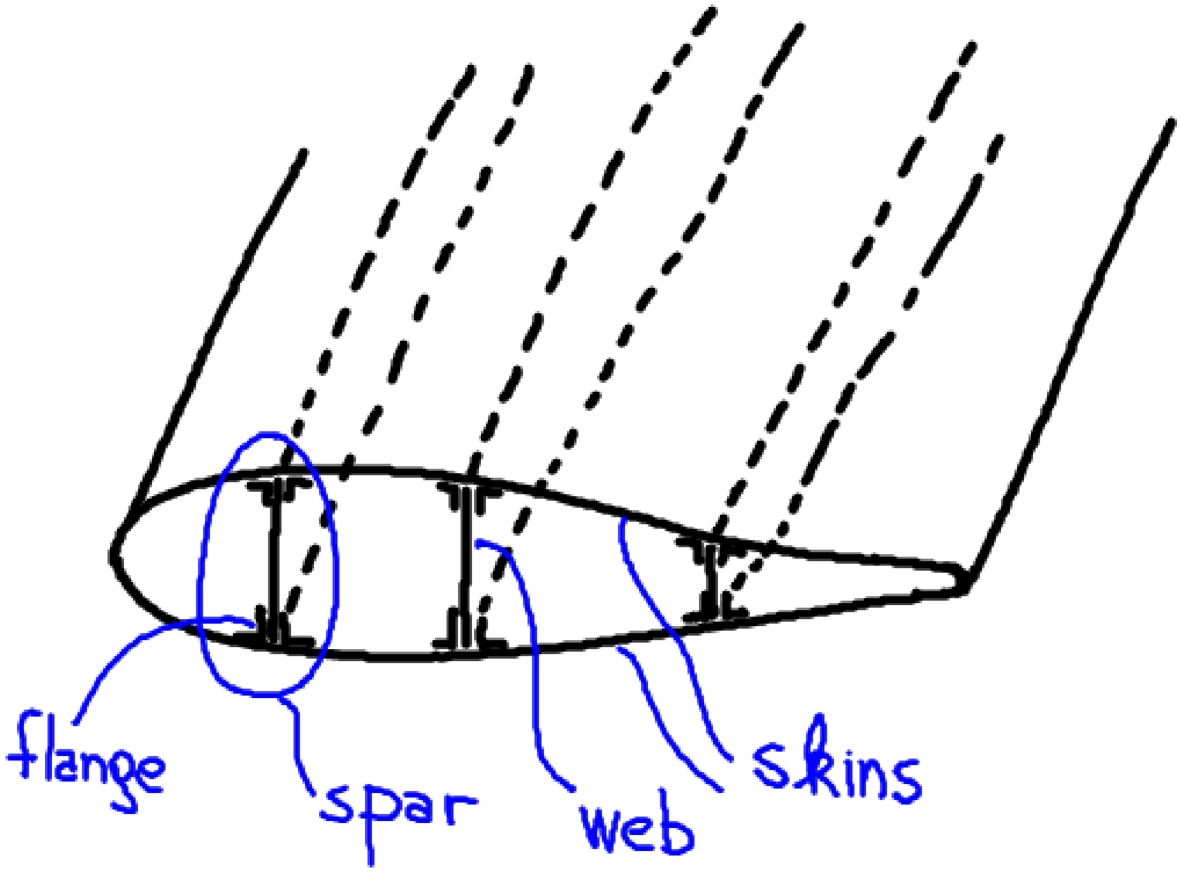


Figure 8.4: Semi-monocoque construction of a wing

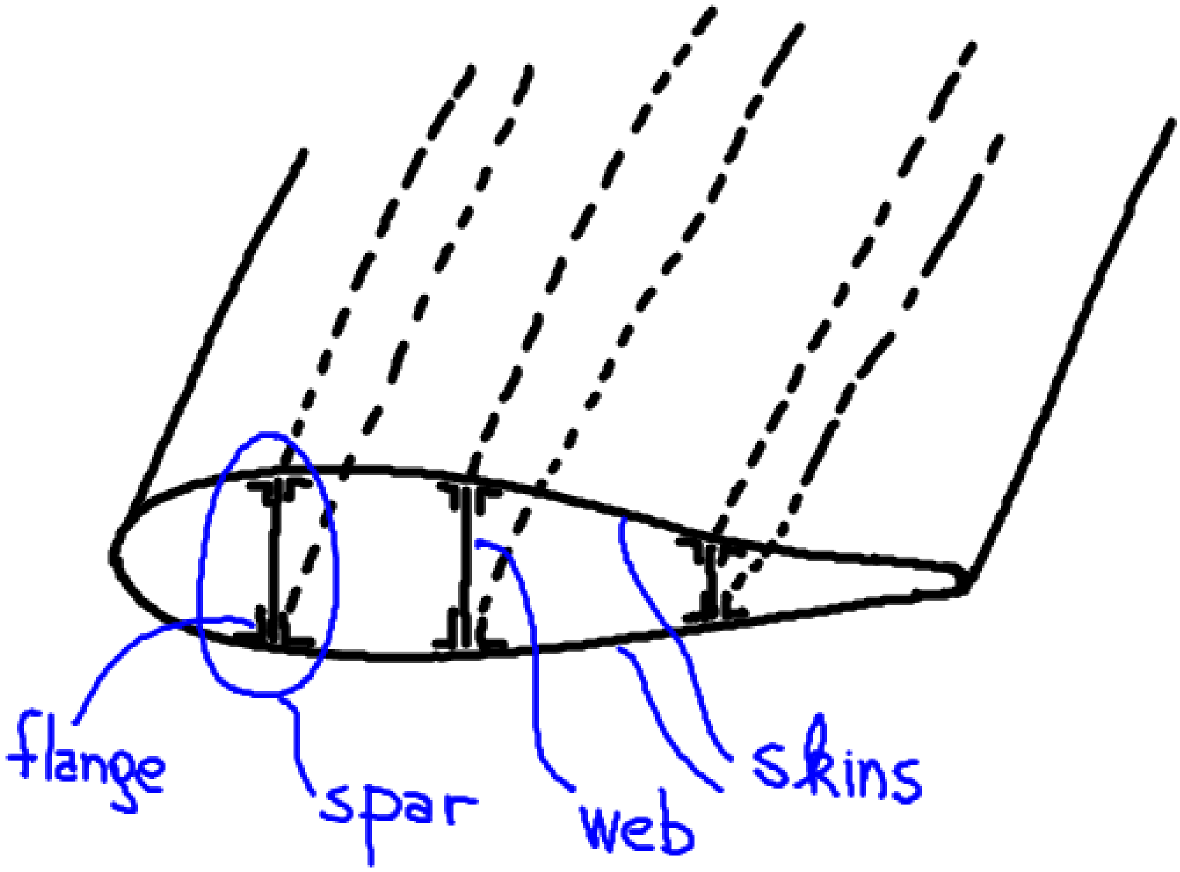


Figure 8.5: Idealization of semi-monocoque structure as a shell beam

We will analyze the wing as a cantilevered beam under combined bending, shear and torsion.

For bending we will assume general beam theory (Euler-Bernouilli hypotheses) with discrete (point) area elements (flanges, stringers) defining the cross-section properties.

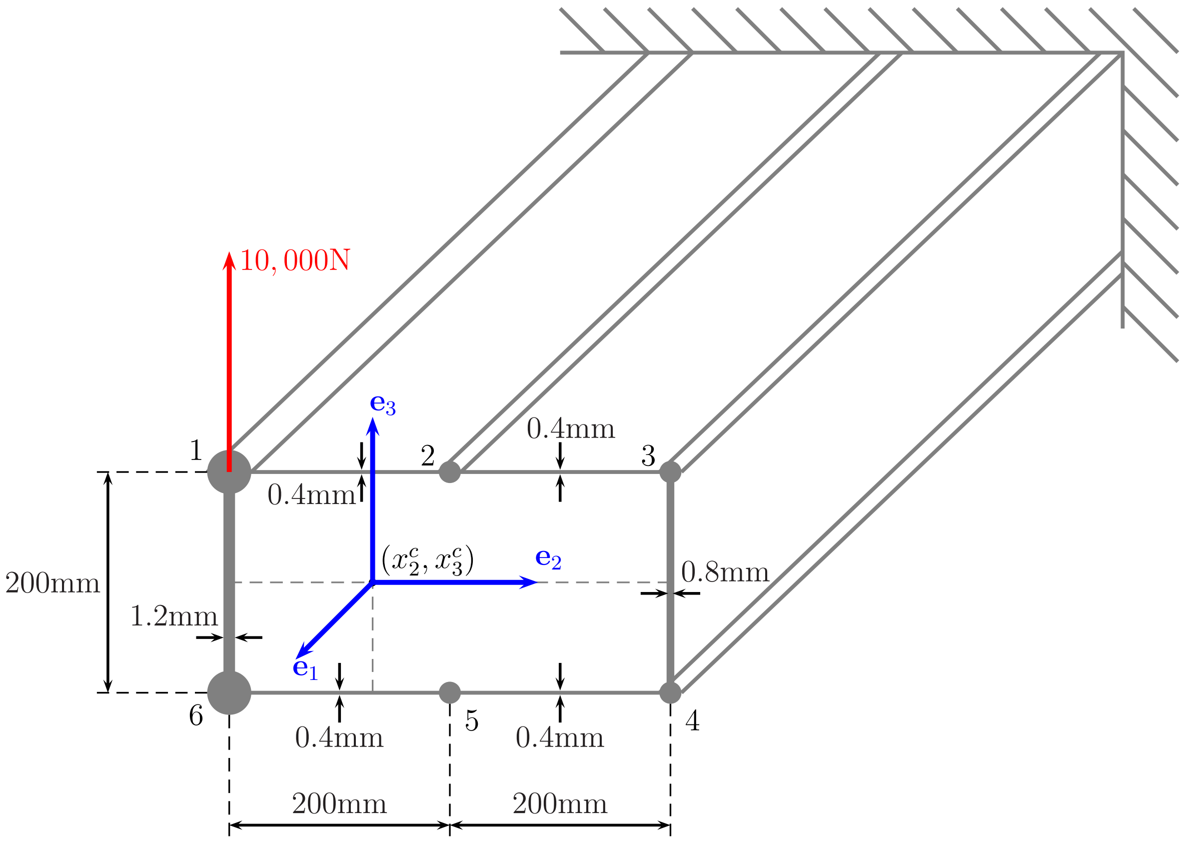
For torsion, we will also assume that the hypotheses of Saint Venant theory are applicable: 1) cross section shape is maintained (this requires rigid ribs space closely enough), 2) cross-section is free to warp our of its plane.

The basic elements of the theory have been developed. The application is rich in details which is best done by example.

8.2.1 Single-cell shell beams

We first consider the case of single-cell beams.

Concept Question 8.2.1. *Example - Single Cell Box-Beam* Let's consider a single cell box-beam of length $l = 2$ m as depicted in Figure 8.2.1. The Young's modulus of the material is E. The beam is clamped at one end (at $x_1 = 0$) and free at another end (at $x_1 = L$) where a concentrated load of $10,000$ N is prescribed at node 1.



The surface areas of the stringers are:

$$A^{(1)} = A^{(6)} = 400\text{mm}^2 \text{ and } A^{(2)} = A^{(3)} = A^{(4)} = A^{(5)} = 200\text{mm}^2$$

The remaining dimensions and skin thicknesses are shown in the figure.

1. Determine the moment M_2, shear force V_3 and torque T distributions along x_1 from equilibrium considerations.

2. Determine the position of the modulus-weighted centroid (x_2^c, x_3^c) (see figure 8.2.1).

3. Determine the axial stress component σ_{11}. For that purpose you will (i) determine the relation between σ_{11} and the deformation ϵ_{11} hence the displacement u_1, then (ii) determine the relation between the displacement u_1 and the axial and bending stiffnesses, and (iii) determine the values of the stiffnesses.

4. *Shear stresses.* Considering that the skin elements of the box beam are very thin, what assumption can we make about the shear stress state through the thickness of any of the skin elements?

5. In order to study the effect of the shearing force in the shell-beam, it is convenient to introduce the notion of the shear flow, f, as we did in torsion theory (at that point we use the symbol q for the shear flow, we are changing it for consistency with Bauchau's book):

$$f^{(i)} = \sigma_{1s}^{(i)} t \tag{8.23}$$

Draw the different shear flows in each of the skin elements on a section of the shell-beam.

6. Write an equation of equilibrium relating the shear flows $f^{(1)}$, $f^{(6)}$ adjacent to joint 1 by considering the variation of the axial stress force $n_1^{(1)} = \sigma_{11}^{(1)} A^{(1)}$ along the axis $\mathbf{e}_1$ (the small caps denotes that this is not the total axial force in the cross section but just on this stiffener).

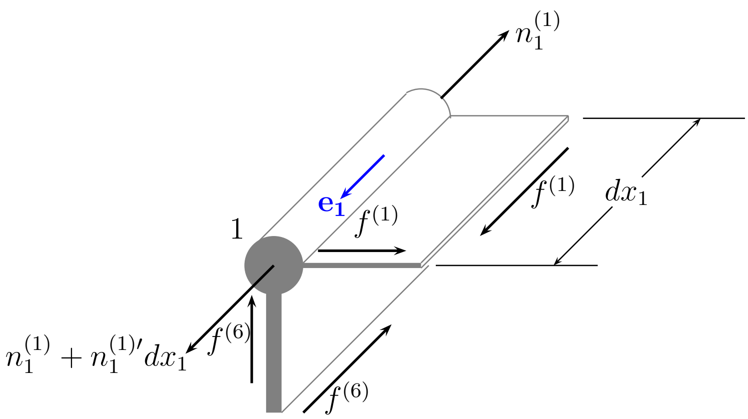


Figure 8.6: Equilibrium of joint 1

7. Show from the previously derived equilibrium equation that a joint equation of the following form:

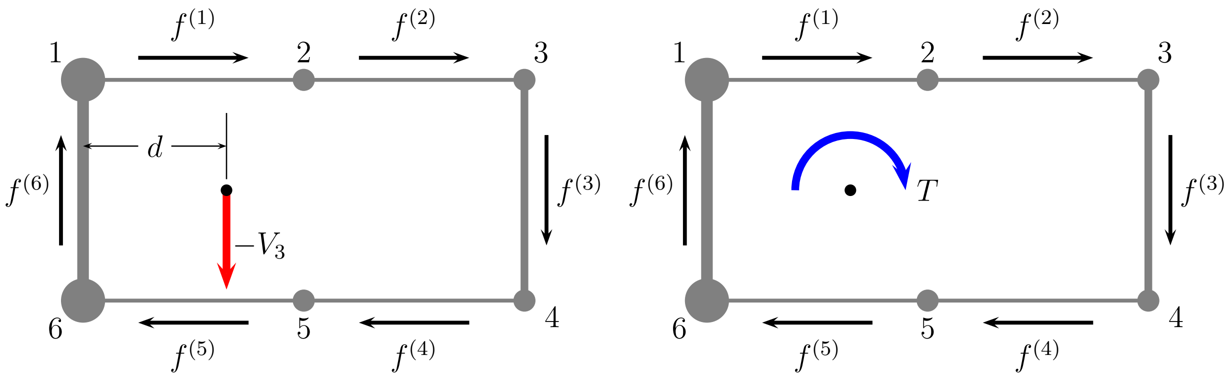
$$f^{(\text{out})} - f^{(\text{in})} = -\frac{Q_2 V_3}{H_{22}^c} \tag{8.24}$$

can be established where $Q_2 = EAx_3$ is the modulus-weighted first moment of area about $\mathbf{e}_2$.

8. Now the shear stresses arise due to reasons:
   - Shear resultant V_2 and V_3
   - Twisting moment T

   It is convenient to break up the analysis into two separate problems:
   - "Pure shear"
   - "Pure torsion"

   The schematic of each problem is illustrated in Figures 8.7(a) and 8.7(b), where d is the distance to the *shear center*.



(a) "Pure Shear" (no twist) (b) "Pure Torsion"

Figure 8.7: Convenient break up of the problem into two separate problems

   Considering $V_3 = 10000\text{N}$ and $T = -10000\text{N} \times d$, write the equilibrium equation for each joint i of the "Pure Shear" problem, Figure 8.7(a).

9. Can you calculate the shear flows $\left\{f^{(i)}\right\}_{i=1,\ldots,6}$ from the above system of equations?

10. An additional equation is obtained by the requirement of *torque equivalence* between the externally applied torque and the internal torque provided by the shear flows. This torque can be computed with respect to any point in the cross section *not* in the shear center, i.e. in the line of application of the load for the first (pure shear problem). Write the general equation and apply it to this problem

11. The last equation to close the system is obtained by imposing the *no twist condition*. From torsion theory for thin closed sections, we obtained that the torque-rate-of-twist relation is given by

$$\oint_{\partial\Omega} \tau ds = 2G\alpha A.$$

   Use this expression to obtain a generic equation to impose the no-twist condition in closed shell-beams:

12. Specialize the no-twist condition to our problem:

13. Solve the system and obtain the six shear flows and the position of the shear center for the case of pure shear.

14. Verify the solution by computing the resulting internal forces and comparing with the external loads

15. If the load were applied at the shear center, there would be no twist of the section. However we still need to deal with the second part, "pure torsion", which results from translating the applied shear force from the point of application to the shear center.

    Now that we know the location of the shear center, compute the torque produced by the applied shear force:

16. Write the equations of joint equilibrium for the case of pure torsion. Can you compute the shear flows from these equations alone? What can you conclude about the shear flows for the pure torsion case?

17. Apply the Torque Boundary Condition and obtain an equation to close the system. Use the fact that all the shear fluxes are the same to solve the system.

18. Draw schematics of the cross section with the numeric values and directions of the shear flows obtained for the pure shear and pure torsion cases, then draw a diagram for the combined flows. Interpret the results.

8.2.2 Multi-cell shell beams

When we have box beams with several close cells, a few different considerations are in order.

Concept Question 8.2.2. Consider the box beam but with an additional web (2 cell wing), as illustrated in Figure 8.8

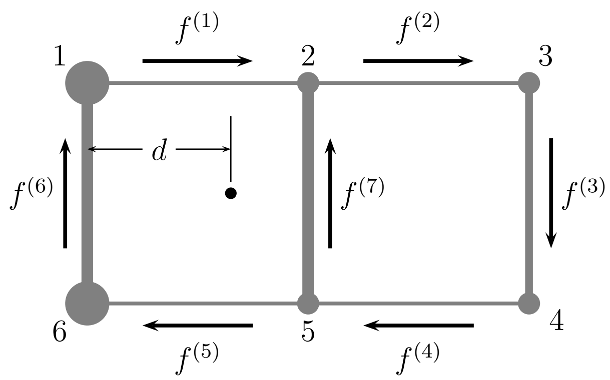


Figure 8.8: Box-beam with two cells

The thickness of the new web is 1.2 mm. Note that since the stringers have not changed, the axial stresses due to bending remain exactly the same. For the analysis of the shear stresses, we follow the ideas introduced in the problem for one cell, and we break up the analysis into two separate problems: "pure shear" and "pure torsion".

1. Write the equilibrium equation for the joints for the "pure shear" problem. How many unknowns and how many independent equations do you obtain?

2. Impose the torque boundary condition to obtain another equation. Are you any closer to solving the system?

3. Apply the *no-twist condition* for the special conditions of this problem. How many equations do you get from this?

4. Solve the system of equations and obtain the shear flows for the pure shear problem. Comment on the shear flow distribution of the two-cell vs the single-cell box beam

5. Now consider the "Pure Torsion" problem, obtain the joint equilibrium equations. How many equations and unknowns do you get?

6. Apply torque boundary condition

7. Since we expect to have twist due to the torque, what other kinematic condition would make sense in this case? Apply it and show that this closes the system of equations

8. Solve the system of equations for the pure torsion problem:

9. Add the shear flows for the pure shear and pure torsion problems to obtain the total shear flows:

10. Sketch the shear flows for both the pure shear and pure torsion cases as well as the combined (full) solution to the problem.

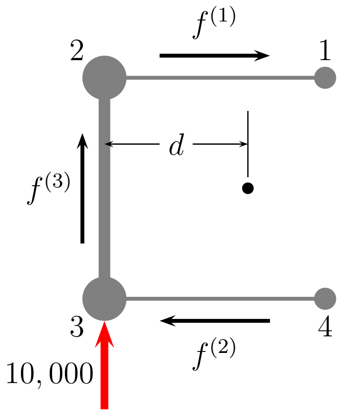
    The same ideas are followed for wings composed by 3 or more cells.

8.2.3 Open-cell shell beams

Concept Question 8.2.3. Consider now an open section as the one illustrated in Figure 8.2.3

The computation of the axial stresses and deflections due to bending or axial loads is done as before.

The computation of the shear stresses requires a few special considerations. Again, we break up the analysis into two separate problems: pure shear and pure torsion.



1. *Computation of the shear fluxes for the pure shear problem:* Again, we assume in this case that the shear force is applied on the shear center so that there is no twist. For the open section of the figure

   Write the joint equilibrium equations and show that the shear fluxes in each skin can solved for from these equations alone. Explain why this is the case.

2. *Computation of the shear center location:* Explain how you can determine this by enforcing torque equivalence.

3. *Solution of the pure torsion problem:* Reflect on the nature of the shear stresses due to torsion in the case of open vs closed cross sections (Remember the membrane analogy?). Draw a sketch of the type of shear stress distribution in this case Does it make sense to talk about shear flows in this case?

4. Why is the open section a bad idea?