V. Lectures 21-24

VA: State Space, Eigenvalues, and alternative representations of dynamical systems (JVV 11.2-11.4)

- Understand the concept of “state”, and it’s significance for modeling dynamic systems and representing them mathematically and for simulation.

- Be able to convert from transfer function (input-output) form to state-space form, and back again. This is particularly important for understanding the invariance of dynamic systems to their representation, and for showing that similarity transformations yield different state-space representations of the same dynamic system.

- Learn the relationship between the characteristic equation in a transfer function, \( \det(I_s - A)^{-1} \), and the eigenvalues of a matrix A in the context of the differential equation \( \dot{x} = Ax + Bu \).

VB: Eigenvalues and Eigenvectors (JVV 11.6)

- Understand how the eigenvalues and eigenvectors of a systems “A” matrix can be used to write the solution to a first order matrix ordinary differential equation \( \dot{x} = Ax + Bu \).

- Be able to use the eigenvalue decomposition of a matrix (whose eigenvalues are distinct) to perform a similarity transformation on a state-space representation, and thereby create a ‘diagonlized form’, in which each equation is independent.

- Use the diagonalized form to understand the importance of the eigenvectors in understanding the response of a system, specifically the concept of eigenvalues as ‘directions’ in state space along which homogeneous responses are constrained.

VC: The Transition Matrix (JVV 11.5)

- Understand the concept of a transition matrix; what it does, what properties derive from its definition.

- Learn various methods for computing and representing the transition matrix.

- Understand the importance of transition matrices in the context of discrete-time systems.
VA: State Space, Eigenvalues, and alternative representations of dynamical systems

- Understand the concept of “state”, and it’s significance for modeling dynamic systems and representing them mathematically and for simulation.

Write the linearized, state space equations for the Quanser apparatus, using the equations provided in Lab 1.

(a) First write the dynamics if the inputs are $\delta \omega_{cyc}$ and $\delta \omega_{coll}$. This will require the use of equations (7)-(9)

(b) Now add the dynamics of the motors, by making $\delta \omega_{cyc}$ and $\delta \omega_{coll}$ into states and introducing equations (10)-(13), so that the inputs become $\delta \tau_{cyc}$ and $\delta \tau_{coll}$

- Be able to convert from transfer function (input-output) form to state-space form, and back again. This is particularly important for understanding the invariance of dynamic systems to their representation, and for showing that similarity transformations yield different state-space representations of the same dynamic system.

Problems 11.1 – 11.7, 11.18 – 11.20

Problem 11.8. For your state space system, include both $r(t)$ (the reference input going into the summing junction on the left) and $d(t)$, the disturbance. For your output, include both $c(t)$, the shaft position (considered the output here), and $v(t)$, the angular velocity of the motor (input to the integrator) in the drawing. Your final state-space system will be two-input, two-output.

- Learn the relationship between the characteristic equation in a transfer function, $\det(I s - A)^{-1}$, and the eigenvalues of a matrix A in the context of the differential equation $\dot{x} = Ax + Bu$

Problems 11.25 and 11.26 – Use Laplace Transform method of Examples 11.5.1&2

VB: Eigenvalues and Eigenvectors

- Understand how the eigenvalues and eigenvectors of a systems “A” matrix can be used to write the solution to a first order matrix ordinary differential equation $\dot{x} = Ax + Bu$.

As discussed in class, the solution to the homogeneous equation $\dot{x} = Ax$ with initial condition $v_i$, is simply $v_i e^{\lambda_i t}$ if $v_i$ is an eigenvector of the $A$ matrix. Use this fact to write the solution for an arbitrary initial condition $x_0$.

(a) First, write the expression for $x_0$ in terms of the eigenvectors $v_i$. When does this expression yield a solution?
(b) Now, imagine that you are expressing the solution as the superposition of solutions, each of which is the solution for an initial condition \( v_i \). Using the result from part (a), you should be able to write the answer.

- Be able to use the eigenvalue decomposition of a matrix (whose eigenvalues are distinct) to perform a similarity transformation on a state-space representation, and thereby create a ‘diagonized form’, in which each equation is independent.

**Problem 11.28, 11.30 (a), (c) and (d), 11.34 (a) and (b)**

(a) Create the Diagonalized form for the following matrices:

\[
\dot{x} = \begin{bmatrix}
-11 & -3 \\
9 & 1
\end{bmatrix} x
\]

(b) Sketch the trajectories of the states for initial conditions of

\[
x_0 = \begin{bmatrix} 1 \\ -3 \end{bmatrix} \text{ and } x_0 = \begin{bmatrix} 3 \\ -3 \end{bmatrix}.
\]

What is the significance of these initial conditions?

- Use the diagonalized form to understand the importance of the eigenvectors in understanding the response of a system, specifically the concept of eigenvalues as ‘directions’ in state space along which homogeneous responses are constrained.

The ‘harmonic oscillator form’ for a complex eigenvalue was introduced in Section 11.6, and takes the form:

\[
\dot{z} = \begin{bmatrix}
\sigma & \omega \\
-\omega & \sigma
\end{bmatrix} z
\]

(a) Find the similarity transformation \( T \) that yields the harmonic oscillator form for the following dynamical system:

\[
\dot{x} = \begin{bmatrix} 23 & 49 \\ -19 & -30 \end{bmatrix} x.
\]

(b) Plot the trajectories of \( x_2 \) vs. \( x_1 \) for the following two initial conditions.

\[
x_0 = \begin{bmatrix} 1 \\ -3 \end{bmatrix} \text{ and } x_0 = \begin{bmatrix} 3 \\ -3 \end{bmatrix}
\]

(c) Convert the initial conditions in part (b) to initial conditions in the \( z \) space, and plot \( z_2 \) vs. \( z_1 \) for these initial conditions (alternatively, you can simply transform your solution to (b) into the ‘z’ space). The special character of
these plots is a clue for part (d)

(d) Based on parts (b) and (c) what would you say is the significance of the
eigenvector for an oscillatory pole?

(use Matlab for parts (b) and (c), using axis(‘equal’) to make sure you see the
shape of the response with no distortion).

VC Transition Matrices

PROBLEMS TO BE ADDED – THIS PORTION WILL NOT BE COVERED IN
THE QUIZ.

- Understand the concept of a transition matrix; what it does, what properties derive
from its definition.

- Learn various methods for computing and representing the transition matrix

- Understand the importance of transition matrices in the context of discrete-time
systems.