Rate Limiting:
1. What is it?
2. Where does it come from?
3. What is the effect?
4. What analysis tools can we apply?

Consider two systems:

(a) \( u_c \xrightarrow{1/Ts+1} u_1 \)
(b) \( u_c \xrightarrow{1/Ts+1} u_2 \)

System (a):

\( n_c = \frac{u_c}{n} \)
\( u_c = 3 \)
\( u_c = 2 \)
\( u_c = 1 \)

Output reaches 63% of input in \( T \) seconds regardless of amplitude of \( u_c \).

\[ u_i = (1 - e^{-t/T})u_c \]
\[ \dot{u}_i = \frac{1}{T} e^{-t/T}u_c \]
\( \dot{u}_{max} = |\dot{u}_i|_{t=0} = \frac{u_c}{T} \) increases as \( u_c \) goes up.

System (b):

\( u_{z, max} = \frac{2}{T} \)

Rise Time Depends on Amplitude of Input! NONLINEAR
1) A rate-limited actuator or device is one whose maximum rate of change or "slew rate" is limited to some maximum value.

2) Rate-limited systems are nonlinear — their behavior depends on the amplitude of their input or motion.

What causes rate limiting?

Usually an amplitude limit in a feedback system.

Example: Electric Servo (e.g., Futaba R/C Servo)

---

Step input on $U_{com}$: $r/w_s$.

$w_s$ goes at max rate for much of the time.

$U$, $U_{com}$: Rate limited until this time.
That's why servo specs are given in terms of slew rate or time to travel 90° etc. → because a bandwidth specification is not appropriate.

What is the effect?

→ **Limits Bandwidth:**

  E.G.: Cannot achieve high frequency, high amplitude motion

  I.E. Aggressive lead compensation may fail.

  N.B. (a linear actuator model will not capture this problem)

Consider sinusoidal input to a rate limiter

**INPUT:**

\[ U_c = A \sin \omega t \]
\[ U_c = A \omega \cos \omega t \]
\[ U_{c_{\text{max}}} = \frac{A \omega}{A} \]

Both amplitude & frequency effect max rate in a sinusoid!

**OUTPUT:**
Even if small-amplitude BW is high, system will "roll off" due to rate limits if amplitude is too large.

Smaller amplitude — follows perfectly $\mu = 1$

Larger amplitude — slightly lower $\mu$, some delay

Larger still: $90^\circ$ max phase lag, amplitude continues to decrease

Rate limit essentially limit bandwidth

Amplitude of input determines effective BW

First-order, Bandlimited System:

\[
\begin{align*}
|W/u_c| &= \frac{A_{uc}/RL}{2^0} \quad 5 \quad 2 \quad 1 \\
&\rightarrow \quad u_c \rightarrow \frac{U_c}{5 + u_c} \quad \text{RL} \quad u_o
\end{align*}
\]
This set of graphs is called a "describing function" → an amplitude-dependent transfer function between a sinusoidal input & the sinusoidal component of the output.

Analysis:
1. Assume appropriate bandwidth for expected input amplitudes
2. Check amplitudes/rates of servos for typical commands
3. Nonlinear simulation (simulink)