Zeroes & Non-Minimum Phase (NMP) Systems

What are poles? Eigenvalues or "characteristic values" for dynamics to settle, oscillation periods, etc. "Self dynamics"

What are zeroes? Essentially, they describe the response to inputs. 

Rise due to different time constants in forced response, interacting input. 

Mathematically, generalized frequencies (exp(s)) where s.s. output is zero.

Let's look at step responses of:

\[
\frac{1}{s/4 + 1} \quad \text{vs.} \quad \frac{s/2 + 1}{s/4 + 1} \quad \text{vs.} \quad \frac{-s/2 + 1}{s/4 + 1}
\]

Note that all T.F.'s above have same pole, same s.s. gain.

\[
\frac{1}{s/4 + 1} = \frac{4}{s+4}
\]

\[
\frac{s/2 + 1}{s/4 + 1} = 2 \left( \frac{s+4}{s+4} - \frac{2}{s+4} \right) = 2 - \frac{4}{s+4}
\]
Error after some time $T$ is the same

But system overshoots the command

$$\frac{1}{4} \dot{y} + y = \frac{1}{2} u + u$$

$\uparrow$ responds to the derivative of the input, as well as the input

causes an "over-reaction"

speeds up response significantly

adds "lead" to system behavior

$$\frac{-\frac{1}{2} + 1}{\frac{1}{4} + 1} = -2 \left(\frac{\frac{1}{5} - 2}{\frac{1}{5} + 4}\right) = -2 \left(\frac{\frac{1}{5} + y}{\frac{1}{5} + y} - \frac{6}{\frac{1}{5} + y}\right)$$

$$= -2 + 3 \cdot \frac{\frac{4}{5}}{\frac{1}{5} + 4}$$

Error significant but larger @ $t = 1$ (3x)

still negative at $t = \frac{1}{4}$!

$\rightarrow$ system responds more slowly

$\rightarrow$ goes wrong way at first = destabilizing

$\rightarrow$ can feedback overcome these problems?
Second Order Example (more typical)

\[ 2 \frac{(s+2)}{s^2+2s+4} \]

\[ 2 \cdot \frac{s^2+2s+4}{s^2+2s+4} = \frac{2s}{s^2+2s+4} + \frac{4}{s^2+2s+4} \]

\[ \approx 0.45 \]

\[ \approx 0.7 \text{ (oscillation)} \]

\[ 0.7 \text{ (overshoot)} \]

\[ 0.95 \]

\[ 5.3 \text{ sec} \]

\[ 0.47 \text{ (rise time)} \]

\[ 6.1 \]

slope @ \( t=0 \neq 0 \Rightarrow \text{faster rise time} \]

settling time \( \approx \text{same} \)

\[ 2 \frac{(-s+2)}{s^2+2s+4} \]

\[ -0.35 \]

\[ 1.3 \]

\[ \approx 7 \text{ sec} \]

1) 
undershoot - destabilizing in a feedback loop
2) slower response \& settling time
3) still get overshoot, too

NM\&E BAD
Example of NMP system

Micro Hovercraft

to translate, tilt vehicle

Initially pushes the wrong way!

Smarter way:

Initial force is in the right direction!
What is the effect of feedback on the zeroes? Now!

\[
\frac{G_c G}{1 + G_c G} = \frac{N/D}{1 + N/D} = \frac{N}{D+N} \quad \text{zeroes do not change}
\]

What about canceling the zeroes? \( \frac{N_c}{D_c} \cdot \frac{N}{D} = \frac{N_c}{D_c} \)

Min Phase: OK, but usually get benefits from zeroes, no need to cancel

NMP: NO! cannot introduce unstable poles and then insure that controller will never stabilize by making them coincide w/ zero!

So we are stuck w/ NMP zeroes

Need to understand design & limitations w/ these in place

Recall: Time delay introduces NMP zeroes!
Asymptotes for NMP zeros

First order

\[ s \]

\[ \frac{a}{5} \]

\[ 5a \]

\[ -90 \]

\[ 40 \text{dB/decade} \]

\[ \text{same as a pole} \]

\[ \text{phase lag w/ high gain = BAD} \]

Second order

\[ s^2 + 2\zeta \omega_n s + \omega_n^2 \]

\[ 40 \text{dB/decade} \]

\[ \text{same as MP} \]

Design principles and approaches do not change

EXCEPT: -20dB/decade at

\[ \omega_c \text{ may no longer provide sufficient roll} \]

(\[ \text{slope / phase relationship is messed up} \]

Design gets much harder)