In root locus, always considered the departure angle of the pole.

Lead control of a lightly damped pole.

Possible departure angles for a pole-zero pair ("dipole").

In the frequency response, these characteristics are conveyed by the phase margin available.

Plenty of PM
Easy to stabilize
(No lags in system)

Again, plenty of PM
Easy to stabilize
(Zero is providing "lag" at the right point)
Some bad cases:

| Physical Causes of Resonance | Ditto here... zero comes too late to help |

- Primary dynamics being controlled (pitch response of A/C)
- Sensors or Activators mounted on flexible structures (flutter in A/C)
- Dutch roll mode in aircraft to bank angle (aircraft) response
- Control of flexible structures (spacecraft)
- Intentionally added filters for rejection of narrow-band noise (helicopters)

No PM for we near resonance
Control Strategies

Lead Compensation

Need two leads (to eliminate > 90° lag)

- Bandwidth of controller must be >> \( \omega_n \)

- Depending on crossover frequency, system BW, and available gain, either \( \frac{1}{s} \) crossover or \( \omega_n \) crossover will dominate CL response

- To reduce high frequency gain, use complex conjugate zeros:

\( \times \)

(Zeroes will show up in response characteristics, so be careful about placement)
Notch Filter

Pole-Zero Cancellation

Cross-over here — this is a reasonable approach.

Cross-over here — need too precise knowledge of dynamics.

Goal:
- Reduce peak magnitude & prevent crossovers
- Improve phase properties
- Need very precise knowledge of resonance
- Always place zero below lowest possible freq of pole (insure phase goes up first)

Notch Filter B

Notch Filters are also used in the presence of external narrowband disturbances:

Assumptions (e.g., rotor freq)

\[ \text{Gain} \rightarrow \text{Gz} \rightarrow S \rightarrow \boxed{\text{Notch}} \]

In both cases, resonance is not improved, it's just made unobservable.
In this case, employ a closely spaced pole-zero pair:

usually employed well above cross-over frequency, so that the phase lag (due to higher pole damping) is not a problem

This type of notch filter should be designed to have no effect on CL behavior → just reject noise
Low Pass Filter

For high-frequency resonances well outside the bandwidth, just make sure TiF has rolled off sufficiently to prevent gain cross-over.

\[ \text{potential instability} \]

\[ \Rightarrow \text{Added GM} \]

This type of filtering is used for robustness to unmodeled dynamics.

Other Possible Solution

- or add mechanical damping (soft mounts)

1. More Sensors or Actuators: Structural dynamic resonances are very sensitive to placement, an advantageous interlacing can sometimes be obtained simply by moving sensor.

2. Ignore / Do Nothing: Natural layout advantageous
   - or - Damping unaffected by control/0K

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