Computer Controlled Systems Summary

1. Must have all elements slow, must model.
2. Must sample fast enough (all dynamics/control within $\omega_s/2 = \frac{2\pi}{T/2}$), and/or apply anti-aliasing filter. Implication for control = lag!
3. If everything happens within $\omega_s/2$, design & analysis in either domain ($s$ or $z$) will work. (equivalent)

\[ y_{\text{sample, 2014}} = \frac{1 - e^{-Ts}}{Ts} \]
- adds 70° phase lag @ $\omega_s/2$
- adds 4 dB attenuation @ $\omega_s/2$
- a pure time delay

4. $D(z)$ can approximate $G_c(s)$ using finite difference equation, with $z^{-1}$ used to denote a one sample delay:

\[ D(z) = \left. G_c(s) \right|_{s=\frac{z}{T}} = \frac{2}{T} \frac{(z-1)}{(z+1)} \]

(aka Tustin’s transform)
\[ D(z) = \frac{a_0 z^2 + a_1 z^1 + a_2}{z^2 + b_1 z^1 + b_2} \]

\[ y_k (z^2 + b_1 z + b_2) = u_k (a_0 z^2 + a_1 z + a_2) \]

\[ y_k (1 + b_1 z^{-1} + b_2 z^{-2}) = u_k (a_0 + a_1 z^{-1} + a_2 z^{-2}) \]

\[ y_k = u_k + u_{k-1} + a_1 u_{k-1} + a_2 u_{k-2} - b_1 y_{k-1} - b_2 y_{k-2} \]

(5) Stability of \( D(z) \) is determined by the roots of the denominator of \( D(z) \) using the unit circle and associated \( b, u_n \) stability etc. rule.

(6) **Design Process:**

(a) Choose \( T \)

(b) Design \( G_c(s) \), keeping in mind limitations of sampler

- Include filtering, delay, sample-and-hold (needed in digital implementation)

(c) Convert \( G_c(s) \) to \( D(z) \); check stability

(d) Analyze \( CL \) and \( CL \) + \( I \)

- Convert to discrete domain - or -
- Analyze in continuous domain
Analysis & Design in the discrete domain will be discussed in the context of state-space methods.

For most SISO applications, it is better to discretize a continuously designed compensator and analyze/verify properties. To analyze, perform O.L.C. or Bode plots of:

\[
\text{Filter} \xrightarrow{D(z)} \frac{1-e^{-Ts}}{Ts} \xrightarrow{G(s)}
\]

\[\text{Let } Z = e^{Ts}\]

Although unwieldy, frequency-by-frequency evaluation is easy for a computer → so just do it!

That's the basics of SISO digital control!

One problem if compensator has several poles, the numerical properties can be poor.

We will launch into state space a little early, so we can deal w/ this problem.