Problem 1:  
(20 pts)  
Write the transfer function that yields the Bode Plot below. This Bode plot is designed in such a way that you can deduce very accurately what all the parameters are (although some of them are not integers!). Therefore you will be graded on the accuracy of your answer, as well as your deductive methods – give your reasoning.

Final slope = 20 dB/decade ⇒ 2 more poles than zeros

Phase = 45° @ \( w = 0.5 \) rad/sec, slope breaks down ⇒ pole @ \( \omega = 0.5 \)

2nd order peak w/ low damping, halfway between 10 @ 20

⇒ need one zero ⇒ min phase @ \( w = 2 \) must be due to a zero:

\[ \text{fig 8.35} \]

min phase = 62° ⇒ \( T_{1}/T_{2} = 16 \), \( \omega_{s} = 8 \), min point @ \( \sqrt{8 \cdot 0.5} = 2 \)

Time Delay = Must have 135° lag @ 1000 rad/sec
more space for Problem 1...

\[ G(s) = 10 \cdot \frac{s/(\omega_2 + 1)}{(s/\omega_1 + 1)(s^2/\omega_n^2 + 2\zeta/\omega_n + 1)} e^{-\tau s} \]

\[ \omega_1 = 0.5 \]
\[ \omega_2 = 8 \]
\[ \omega_n = \sqrt{200} \]
\[ \zeta = 0.1 \]

(based on JW Fig 7.13) either the mag or phase part) (continue on back)

\[ T_d = \frac{13.5 \cdot \pi/180}{1000} \]

Problem 2:
(16 pts)
If the figure in Problem 1 is the loop gain function for a unity negative feedback system:

What is system's phase margin? (4 pts) \( 0^\circ \)

If the gain is increased, at what approximate frequency would you expect the closed loop system to go unstable? (4 pts)

\( 18 \text{ rad/sec} \)

Sketch the step response of the closed loop system. (4 pts)

What type of compensation would you introduce to improve the phase margin of this system without sacrificing closed-loop bandwidth? (4 pts)

lead compensation
Problem 3:  
(20 pts)  Consider a system whose dynamics are given by
\[
G(s) = \frac{1}{(s/3 + 1)(s/30 + 1)}
\]

Design a lag compensator that results in a closed loop system that meets the following specifications:

a) NO LESS THAN 53 degrees phase margin
b) Tracks signals with frequency content up to 0.1 rad/sec with LESS THAN 1% error.

Modify the attached figure to represent KG(s), and sketch in G_c(s)G(s). Your answer should look similar to Figure 8.12 in JVV. Make sure you label your 0dB line, as well as the frequency, amplitude, and phase axes of your plot. Write the complete transfer function of your compensator below.

1) K > 100 to meet reg't (b)
2) choose \( \omega_c \) frequency where \( PM = 60^\circ \)
3) draw 0dB line \( \omega_c \) 40 dB line
4) draw -20 dB/decade line starting at \( \omega_c \)
5) This is good enough, but increase \( K \) to 43 dB to account for asymptotic approximation

\[
G_c = \frac{141}{(s/15 + 1)} \frac{1}{(s/1 + 1)}
\]

(continue on back)
USE THIS PLOT FOR THE SOLUTION TO PROBLEM 3.

NOTE THAT HORIZONTAL LINES ARE ONLY FOR REFERENCE – THE 0DB LINE HAS NOT YET BEEN DRAWN ON THIS FIGURE!
Problem 4:
(20 pts) Design a lead compensator for Problem 3. You can use the plot on the next page as an aid, but you will not be graded on your Bode plot for this problem.

Draw 0dB line 40 dB below low freq asymptote. First pole is well above 1 ⇒ no more gain needed

PM=30° ⇒ need (23° + at least 70), call it 35°

From fig 8.3, a = 4 w_c = w where mag = -10log4

=-6dB

From sketch, w_c = 120 rad/sec

\[
\frac{w_c}{\omega_a} = \frac{1}{T_1} = 60 \text{ rad/sec}
\]

\[
w_c \cdot \omega_a = \frac{1}{T_2} = 240 \text{ rad/sec}
\]

\[
G_c = \frac{5/60 + 1}{240 + 1}
\]

Problem 5:
(10 pts) Discussion of Problems 3 and 4:

(a) Explain the benefits and pitfalls of your two compensators. (5 pts)

lag: + does not require fast servos
- results in a slow system
- some possibility of wind-up
- barely meets low freq spec
lead: - very high gain; needs fast servos

(b) Describe the conditions (e.g. additional requirements or limitations) under which you would choose one compensator over the other. (5 pts)

+ results in very fast system
- good chance that servos will not limit
- tracking error is low up to 2 rad/sec
- potential noise amplification problems.
- If there is some desire for a fast response, lead would be indicated
- If servos are rate limited, lead may be too aggressive
- If magnitude limits are a concern, lag may be a problem
- Lag-lead may be best compromise
USE MAY USE THIS PLOT AS A GRAPHICAL AID FOR THE SOLUTION TO PROBLEM 4.

NOTE THAT HORIZONTAL LINES ARE ONLY FOR REFERENCE – THE 0DB LINE HAS NOT YET BEEN DRAWN ON THIS FIGURE!
Problem 6: 
(15 pts) The specifications for the elevator servo on your aircraft guarantee that it can travel 90 deg/sec (in other words, this is the rate limit or maximum 'slew rate' of the servo). Up to what frequency do you expect the servo to behave linearly, if the servo travels between -15 deg and 15 degrees to move the elevator through its full range of motion?

\[ u_c = 15 \sin(wt) \text{ deg} \]

\[ \frac{du_c}{dt} = 15w \cos(wt) \text{ deg/sec} \]

max rate = 15w deg/sec (w in rad/sec)

linear behavior should persist up to 6 rad/sec