16.31 Fall 2005
Lecture Mon 17-Oct-05 ver 1.0

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TODAY

- TODAY:
  - Stability
- TAKE AWAY:
  - Definitions of stability that are accepted in the field.
  - Stability is the motivation for state feedback control!

NEXT

- NEXT:
  - Lyapunov stability
  - Kalman's Canonical Decomposition
  - Controller and Observer Canonical Forms, & Minimal Realizations
  - Full state feedback & Observers
  - LQR (Linear Quadratic Regulator)
  - Kalman Filter
  - Robustness & Performance Limitations

Internal Stability - Bélanger

DEFINITION:
An LTI system is **internally stable** if the zero-input solution x(t) converges to zero, for any initial state. (Bélanger, p. 105)

$$\lim_{t \to \infty} x(t) = e^{At} x(0) \;\; \to 0$$

- What does this definition of stability preclude?

THEOREM 3.6:
An LTI system is internally stable if, and only if, all eigenvalues of A lie in the open left-half (complex) plane. (Bélanger, p. 105)

PROOF:
Internally stable implies eigenvalues in open left-half plane.

- The terms of the zero-input solution $x(t)$ are of the form $t^k e^{\lambda t}$, where λ is an eigenvalue of A. If all the eigenvalues have negative real parts, then all term of $x(t)$ tend to zero. This shows sufficiency.

Eigenvalues not in open left-half plane implies not internally stable.

- Let one eigenvalue of A, λ, be such that $Re(\lambda) \geq 0$. Let v be an eigenvector corresponding to λ.If the initial state is v, then $x(t) = ve^{\lambda t}$. Since $Re(\lambda) \geq 0$, $x(t)$ does not tend to zero. This establishes necessity.

Stable and Unstable Subspaces

- The eigenvalues with negative real parts are the stable modes, those with real parts that are positive (or zero!) are unstable modes.
- The stable and unstable modes form subspaces, that is, the state space decomposes into invariant stable and unstable spaces.
- In fact, the state space decomposes into three subspaces: the stable manifold, the unstable manifold, and the center manifold.
- The modes with eigenvalues on the $j\omega$ axis are part of the center manifold.
- These concepts hold for nonlinear systems, too.

Consider the following non-linear autonomous system:

$$\dot{x} = f(x,t) \quad f(0,t) = 0$$

DEFINITION:
The equilibrium of the above system is **stable** if, given any $\epsilon > 0$, there exists a $\delta(\epsilon, t_0) > 0$, such that $||x_0|| < \delta$ implies $||x(t; x_0, t_0)|| < \epsilon$ for all $t \geq t_0$. (DeRusso, *et al.*, p. 471)

- Is this stability definition an improvement?

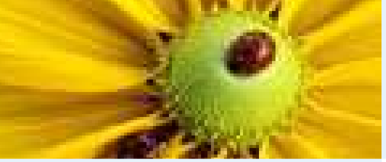
DEFINITION:
The equilibrium is **unstable** if it is not stable. (DeRusso, *et al.*, p. 471)

Asymptotic Stability - DeRusso, et al.

Consider the following non-linear autonomous system:

$$\dot{x} = f(x,t) \quad f(0,t) = 0$$

DEFINITION:
The equilibrium of the above system is **asymptotically stable** if, in addition to the equilibrium being stable, there exists a $\delta_0(\epsilon, t_0) > 0$, such that, if $||x_0|| < \delta_0$, the solution $||x(t; x_0, t_0)||$ approaches 0 as t approaches infinity. (DeRusso, *et al.*, p. 472)



DESIGN

- Often feedback control can provide freedom in the design phase of a system.
- Control can allow greater freedom to achieve performance objectives.
- But there is always a cost!

NEXT

- Wednesday - Lyapunov Stability
- Friday - ???